Algorithms for graph modification problems: towards generality and efficiency

Laure Morelle

September 23rd, 2025

Committee

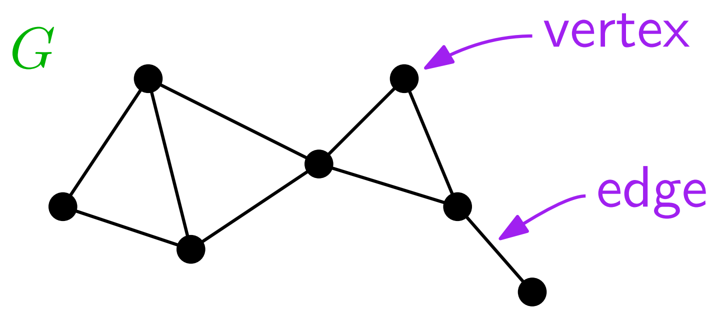
Robert Ganian	Reviewer
Eunjung Kim	Reviewer
Archontia Giannopoulou	Examiner
Petr Golovach	Examiner
Frédéric Havet	Examiner
Ignasi Sau	Supervisor
Dimitrios M. Thilikos	Supervisor

Graphs and Algorithms

Our research:

Design fast **algorithms** to solve computational problems.

Model of abstraction: **graphs**



$V(G)$ = set of vertices of G
$E(G)$ = set of edges of G

Graph modification problems

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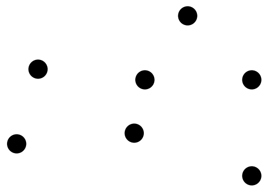

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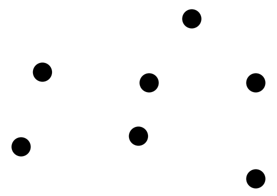
2. A set $\mathcal{M}$ of allowed **graph modifications**



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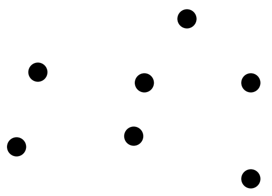

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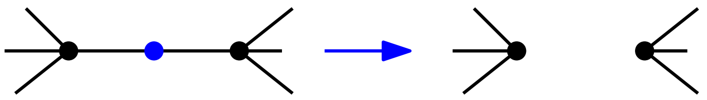

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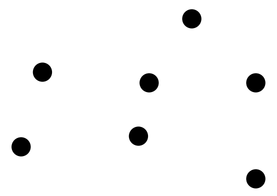

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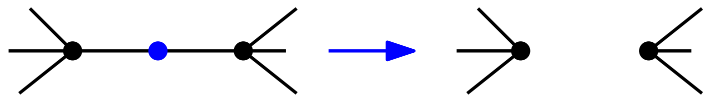

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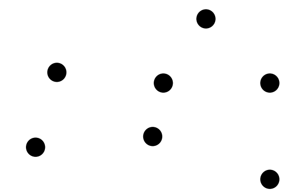

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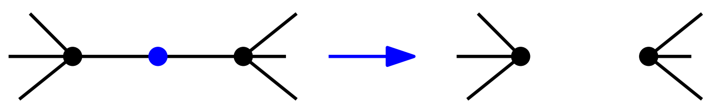

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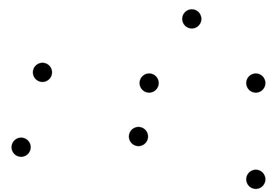

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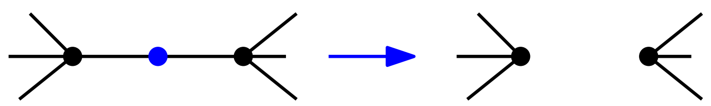

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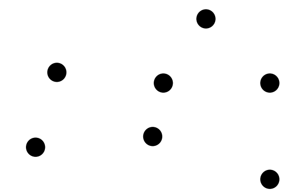

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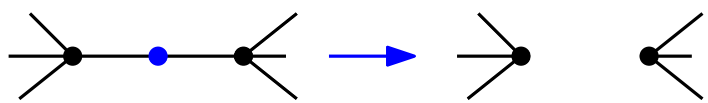

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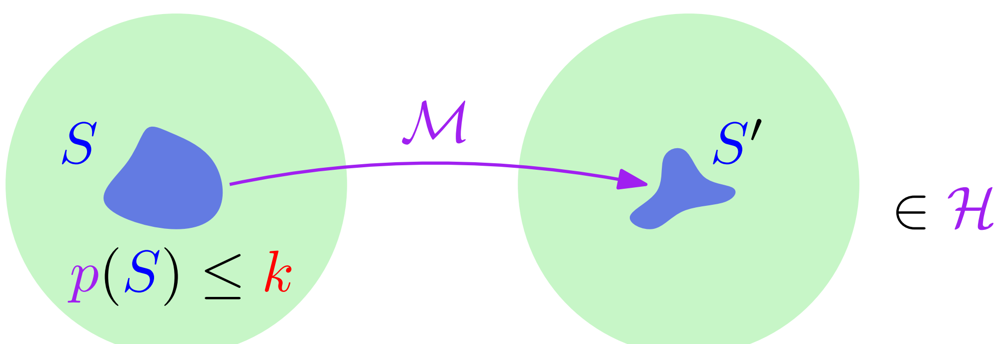

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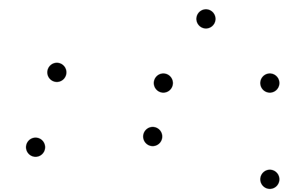

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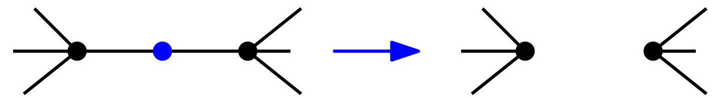

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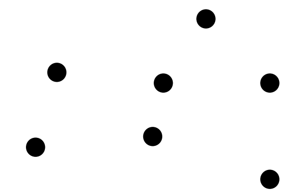

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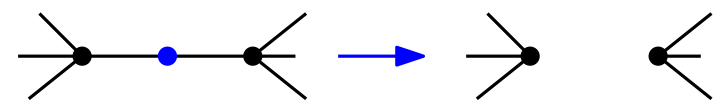

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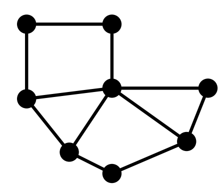

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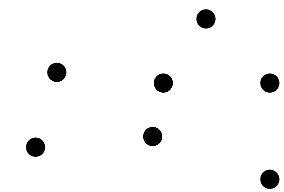

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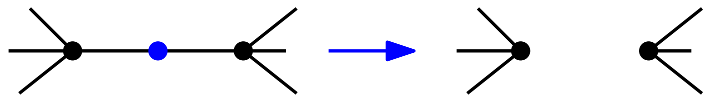

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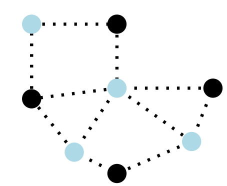

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[Chen, Kanj, Jia, '06]

There is an algorithm solving Vertex Cover in time $\mathcal{O}(1.2738^k + k \cdot n)$.

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Organization

1. Target class
2. Set of modifications
3. Measure on the modulator
4. Vertex identification

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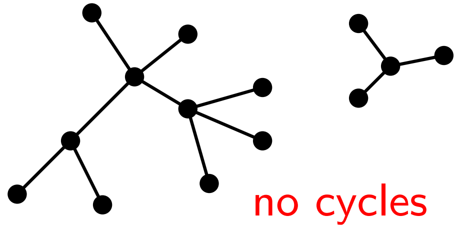

→ Feedback Vertex Set
[Li, Nederlof, '22]
solvable in time $\mathcal{O}(2.7^k \cdot n)$

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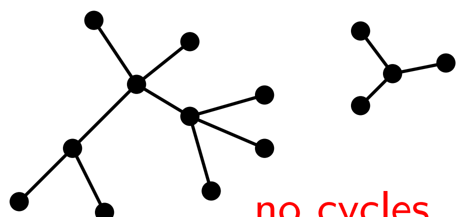

no cycles

$\longrightarrow$ FEEDBACK VERTEX SET
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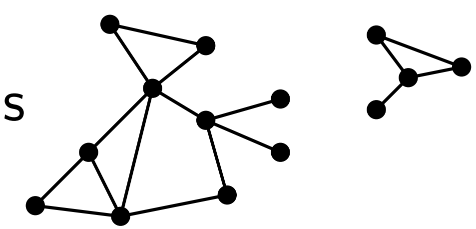
$\subseteq$

$\mathcal{H}$ $=$ planar graphs



$\longrightarrow$ PLANARIZATION
[Jansen, Lokshtanov, Saurabh, '14]
solvable in time $2^{\mathcal{O}(k \log k)} \cdot n$

embeddable on the sphere $=$
can be drawn on the sphere with no edges crossing

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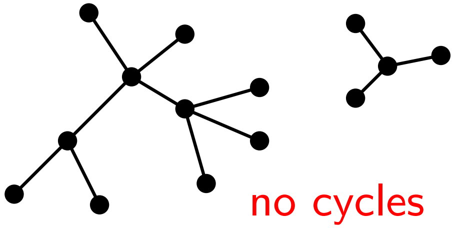

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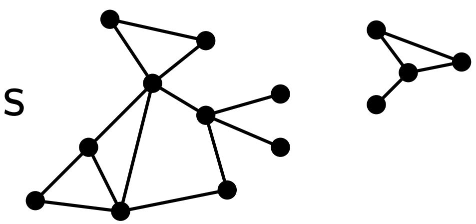

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$\mathcal{H} =$ graphs embeddable on the surface Σ → [Kociumaka, Pilipzuk, '19]
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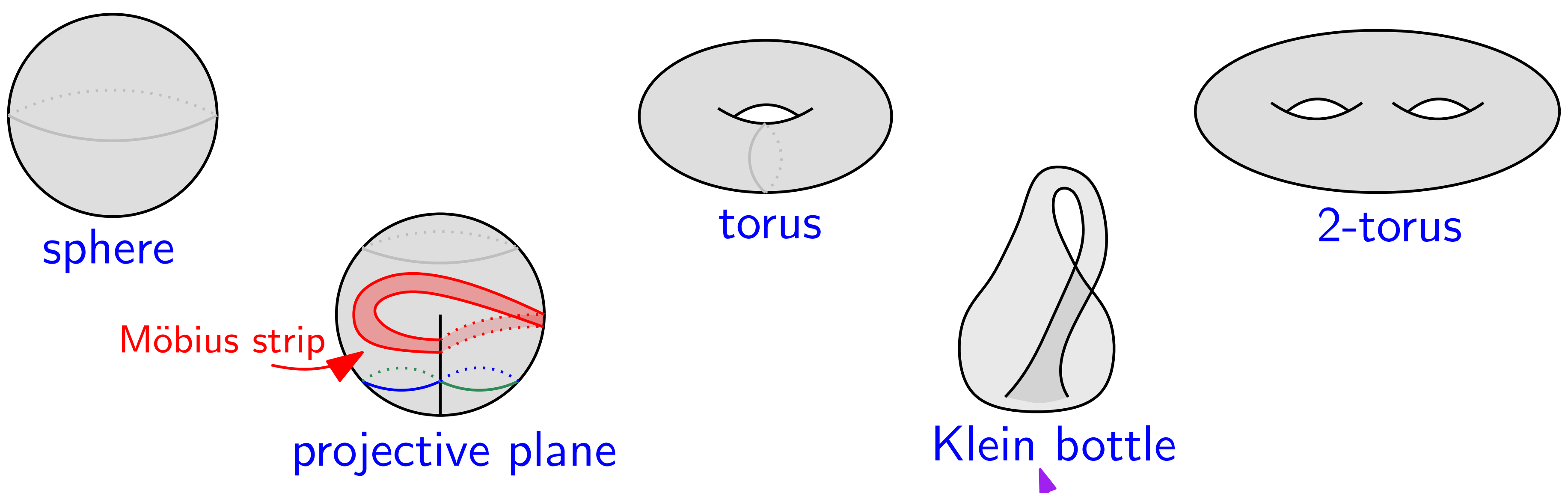

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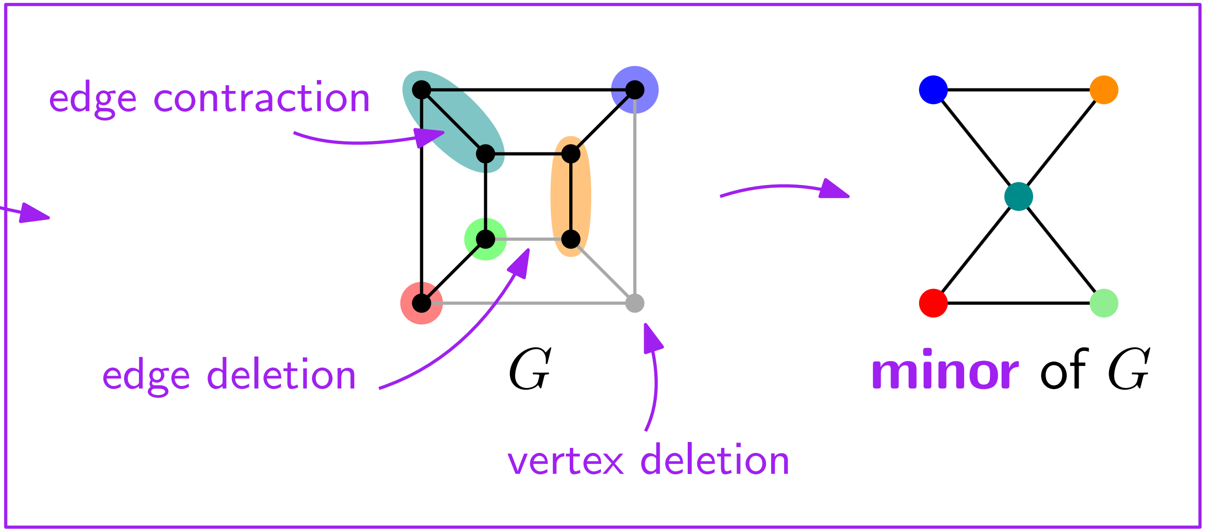

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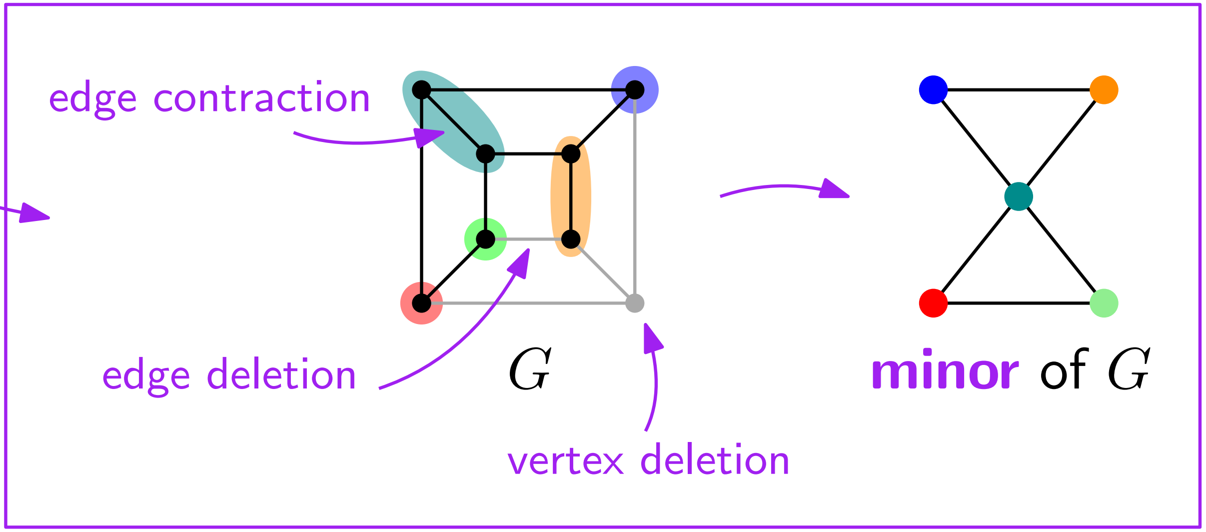

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solvable in time $f(k) \cdot n^2$ for some computable function f

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Sketch of the proof

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Sketch of the proof

originates from [Robertson, Seymour, '95]

Win/Win strategy on the **treewidth** $\mathrm{tw}(G)$ of G:

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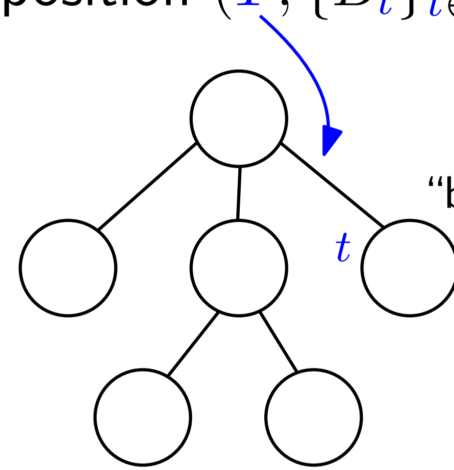
Tree decomposition $(T, \{B_t\}_{t \in V(T)})$ of G



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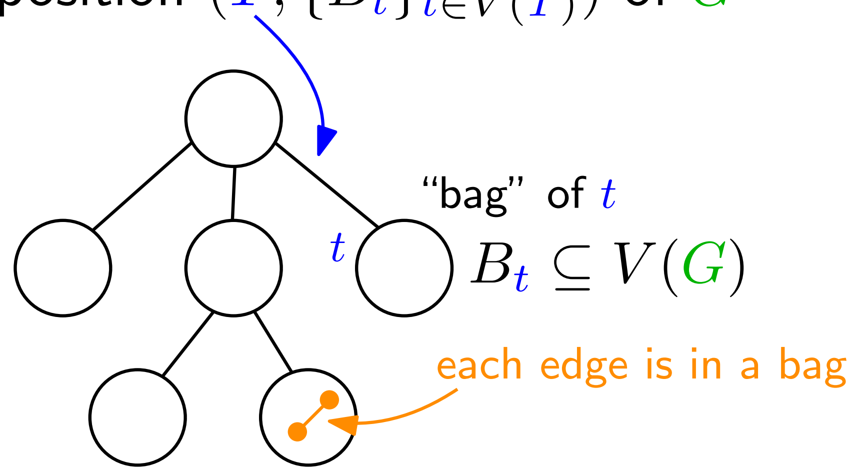

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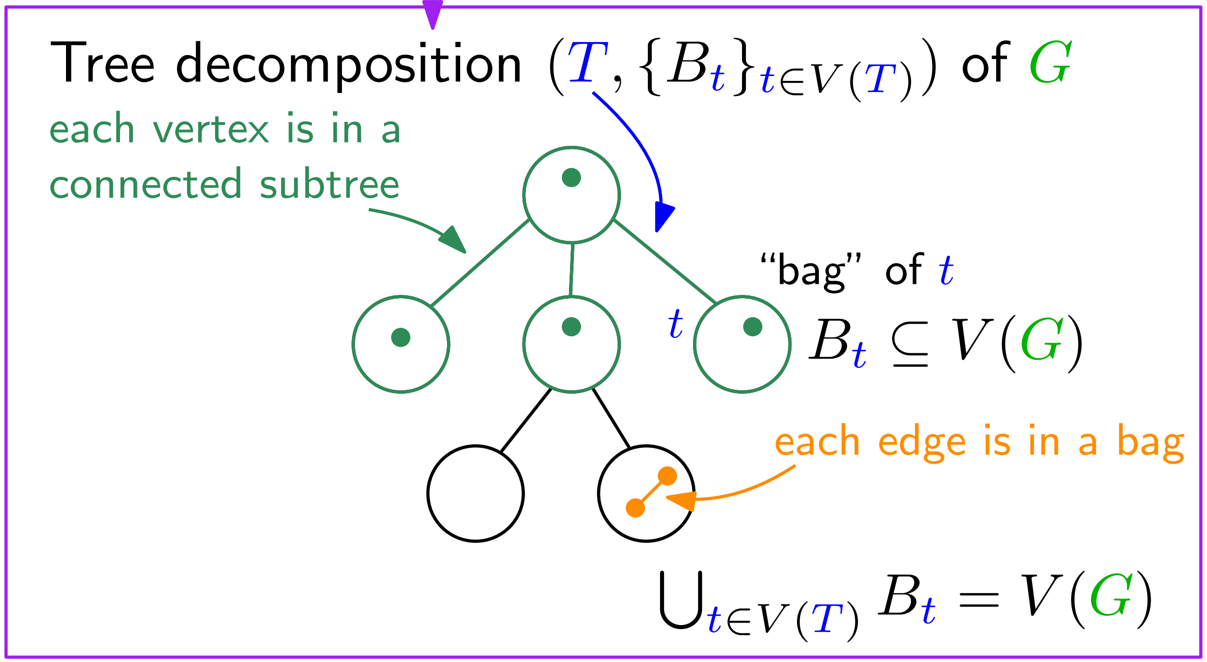

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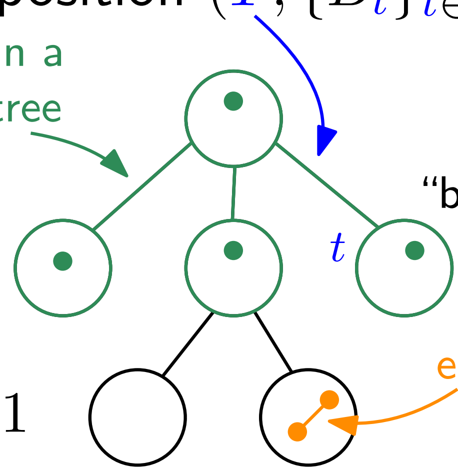
each vertex is in a connected subtree



"bag" of t

$B_t \subseteq V(G)$

tw(G) $\leq k$:

$|B_t| \leq k + 1$

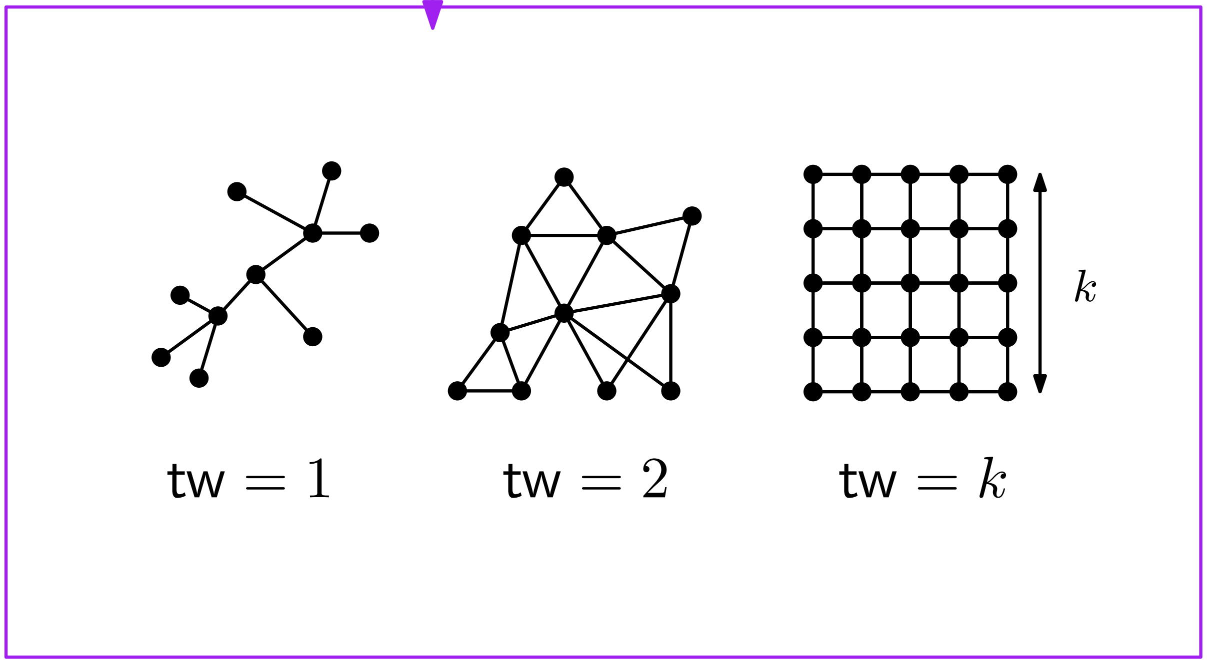
each edge is in a bag

$\bigcup_{t \in V(T)} B_t = V(G)$

Target graph class $\mathcal{H}$

Sketch of the proof

Win/Win strategy on the **treewidth** $\text{tw}(G)$ of G:



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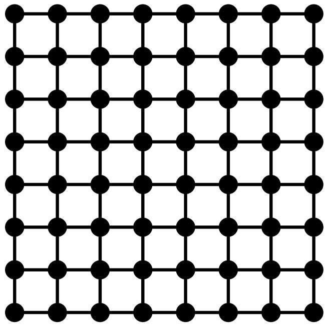

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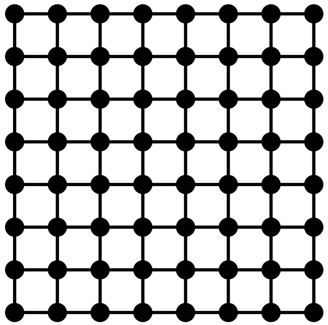

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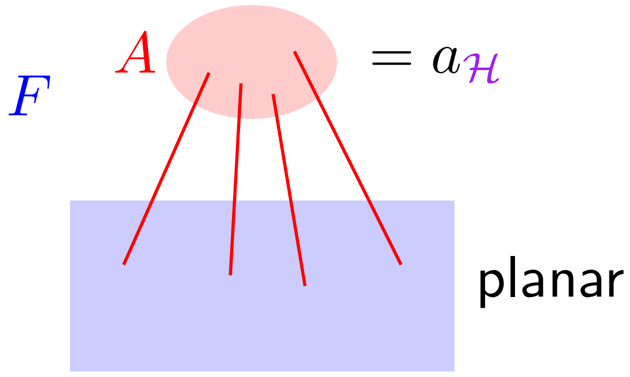

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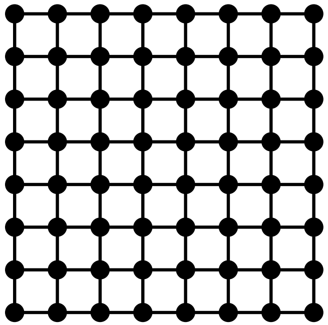

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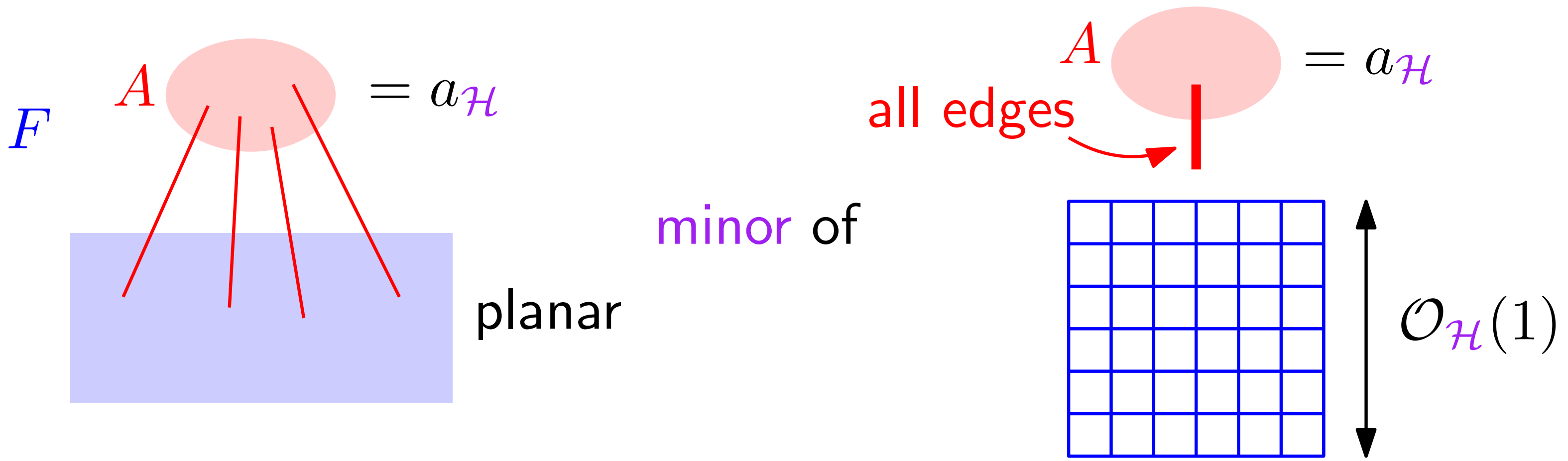

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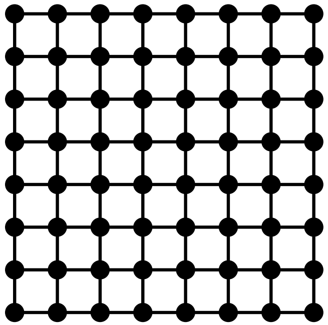

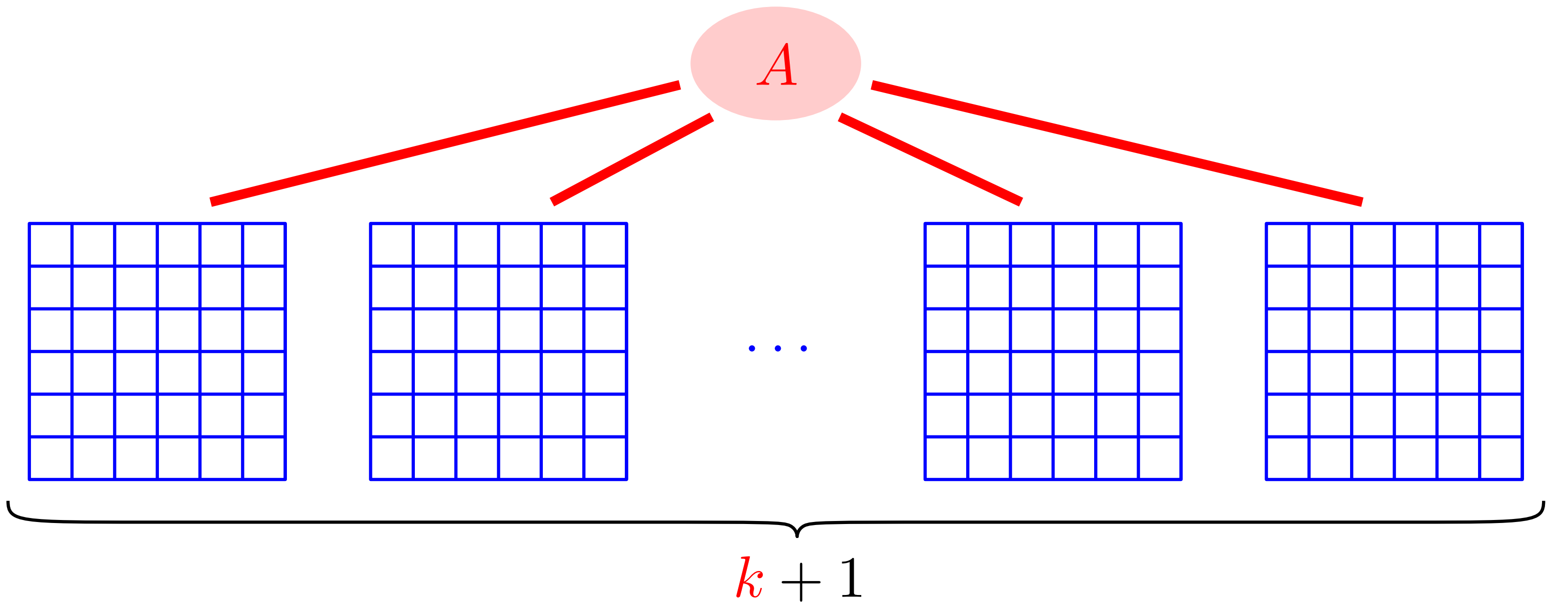

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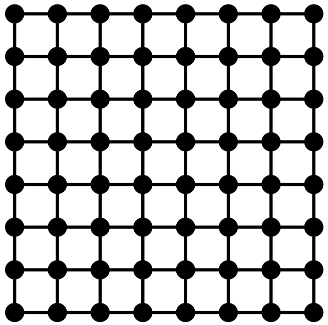

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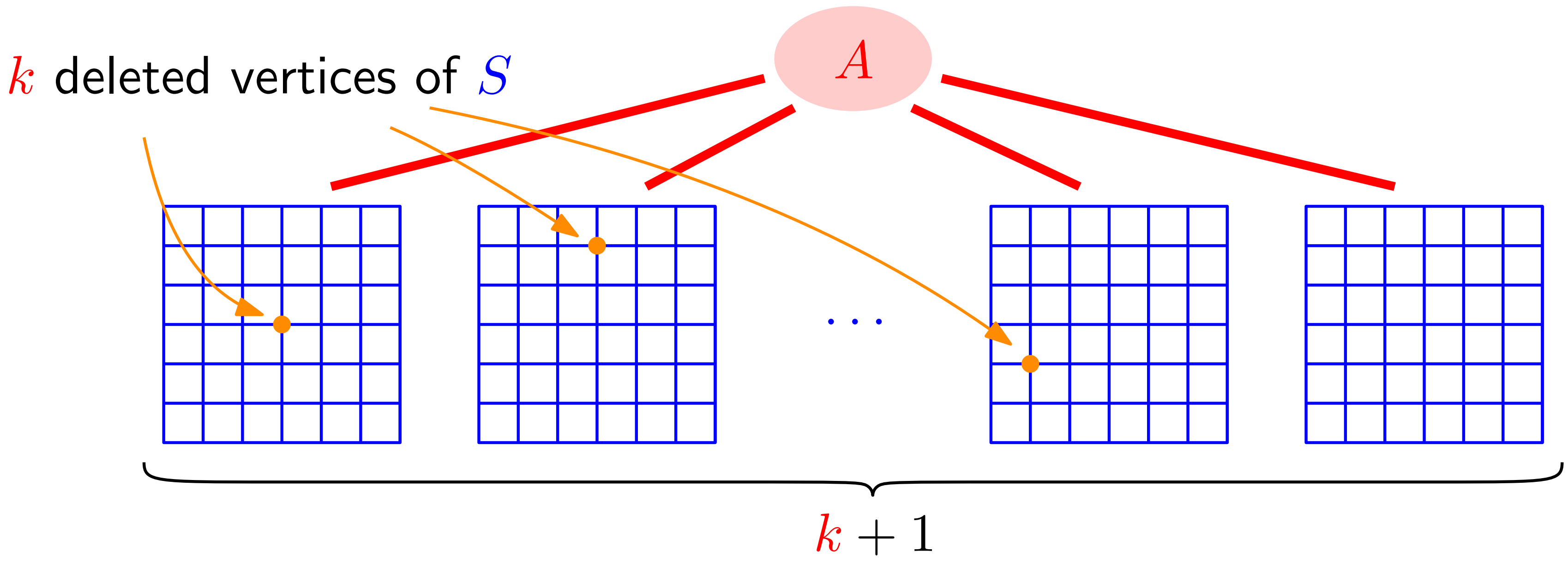

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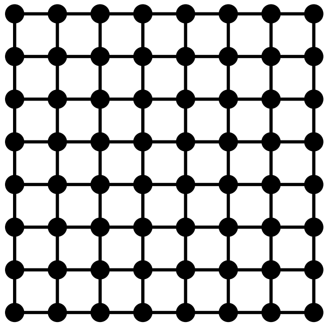

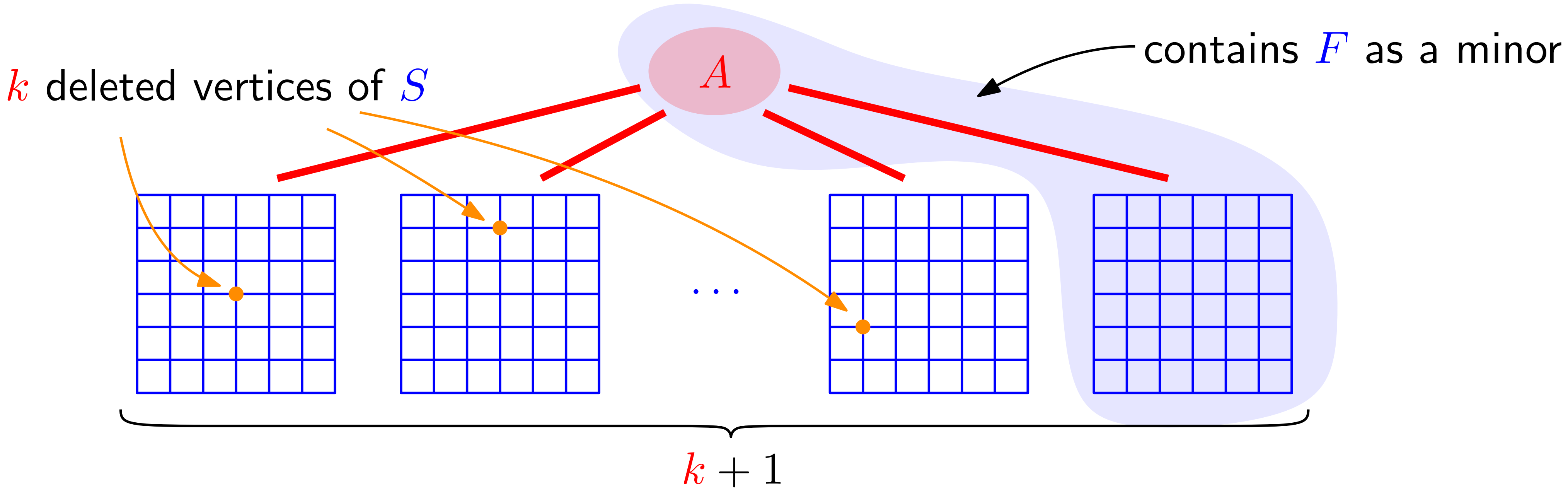

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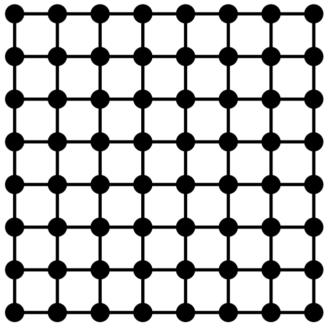

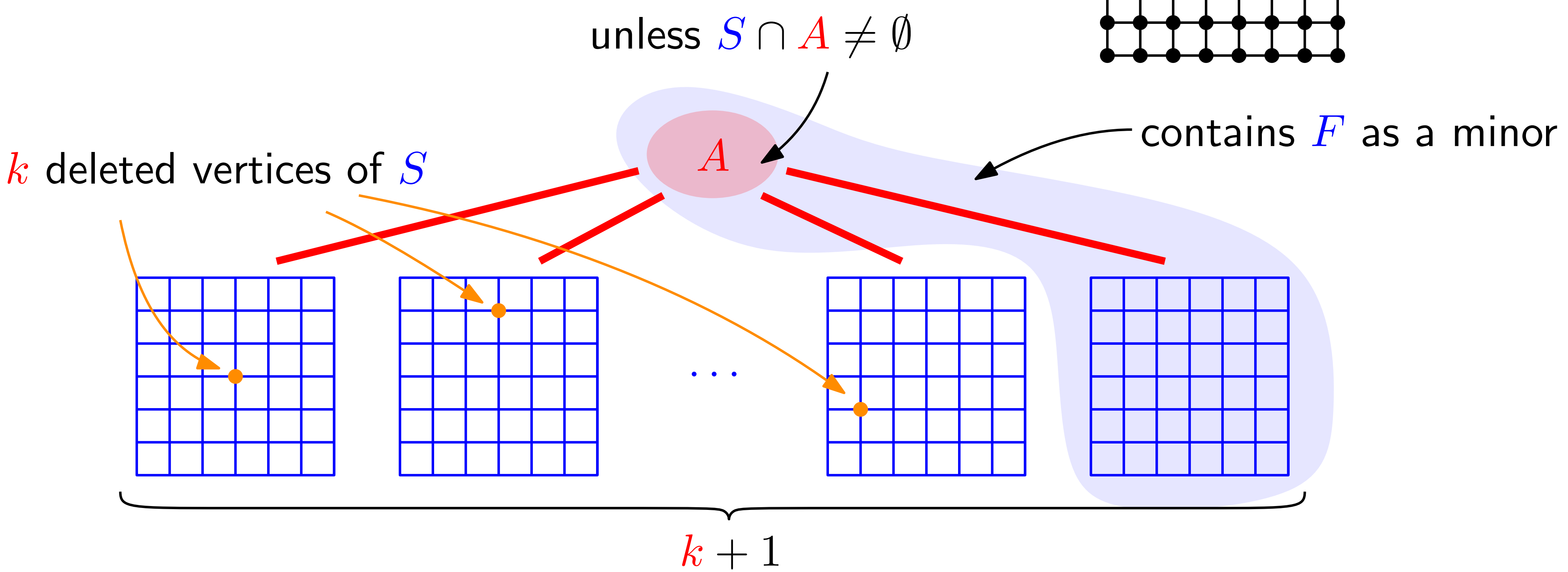

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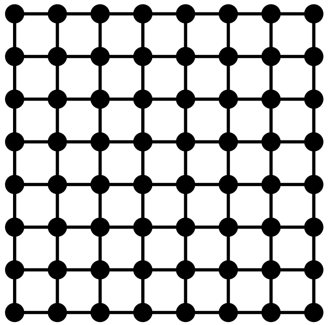

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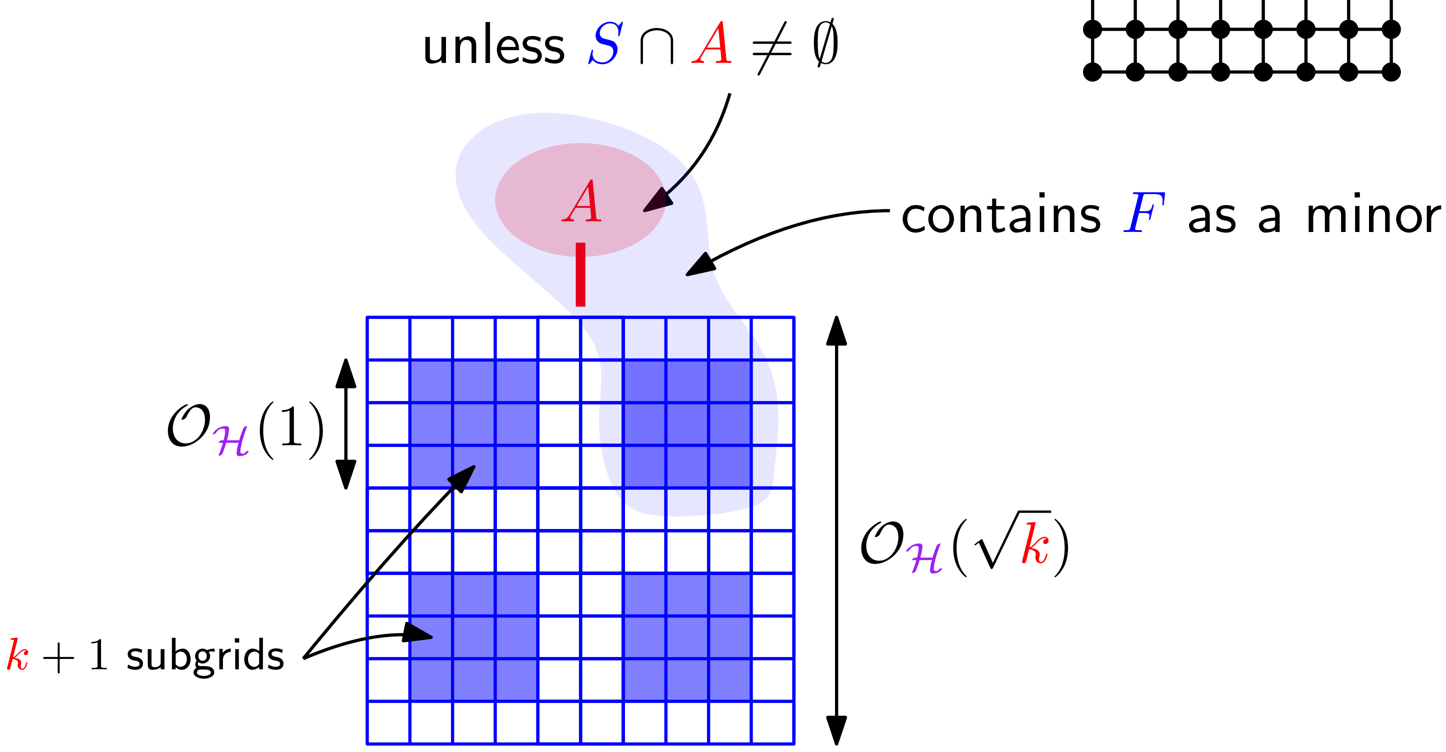

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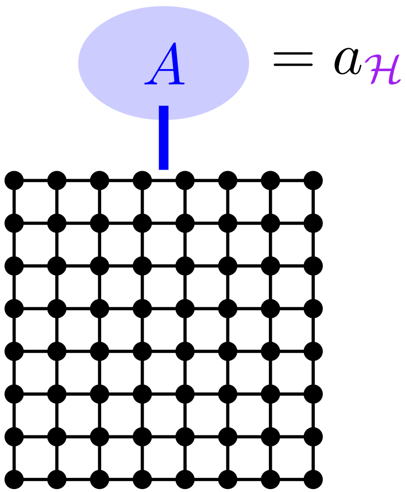

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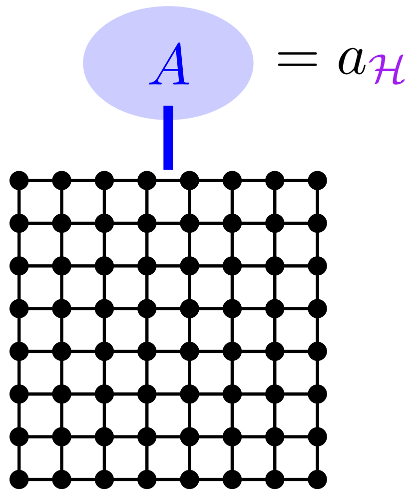

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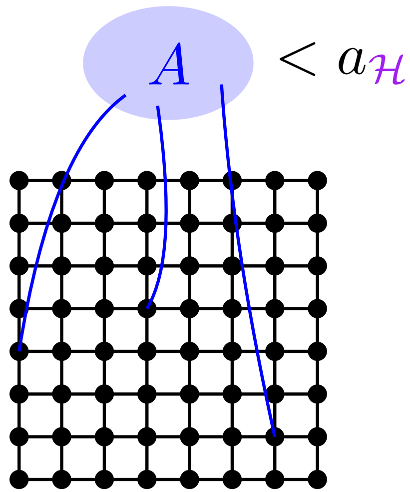

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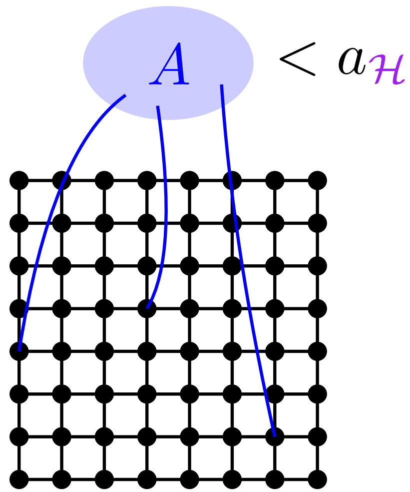

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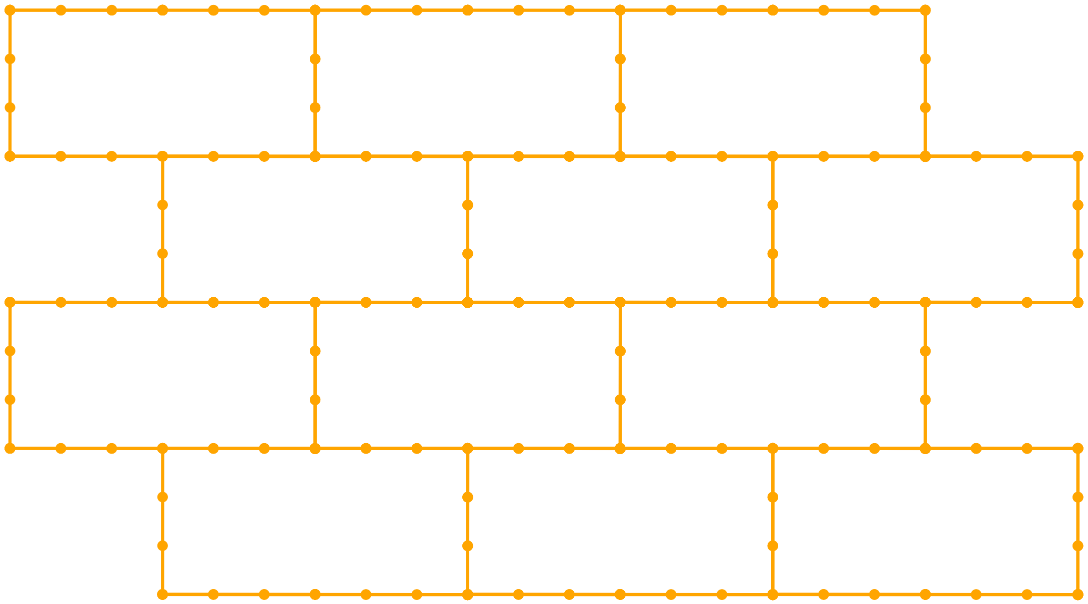

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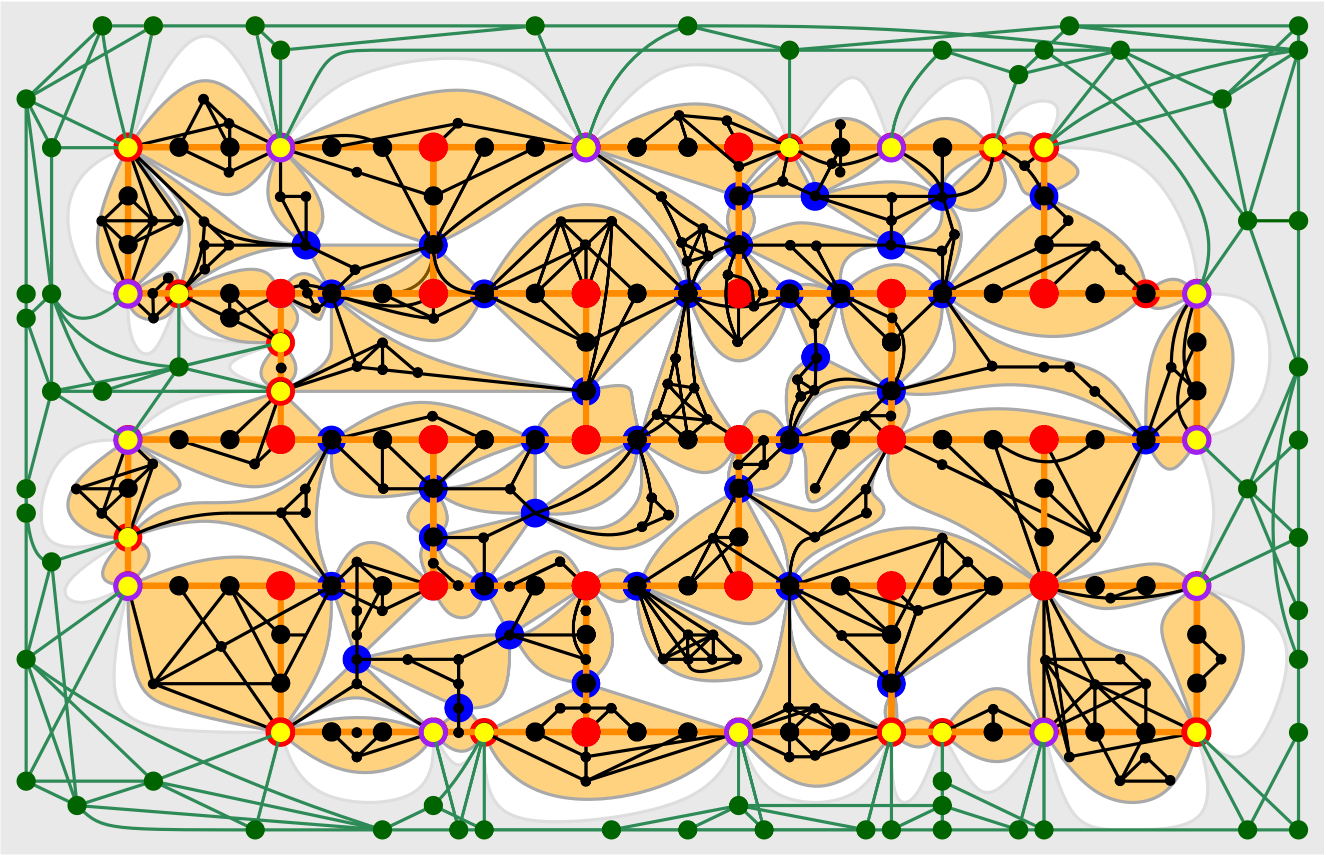

[figure by Dimitrios M. Thilikos]

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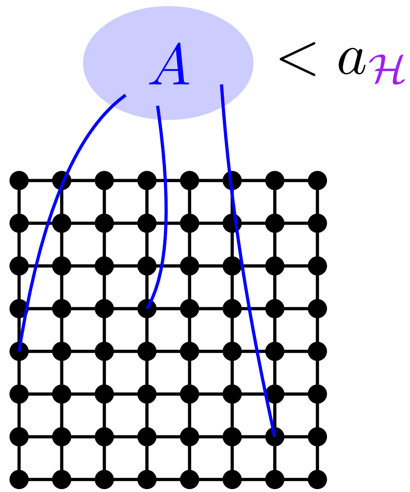

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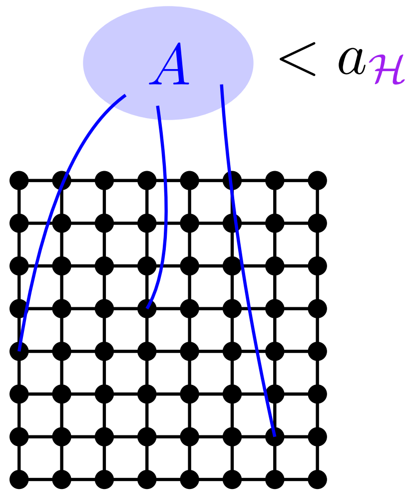

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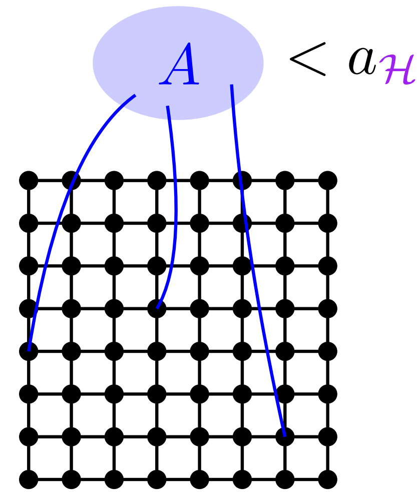

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Representative-based technique [Baste, Sau, Thilikos, '19]

2. Set of modifications $\mathcal{M}$

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Model of abstraction to represent many modifications at once?

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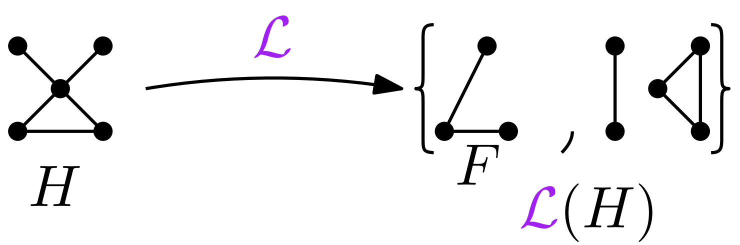

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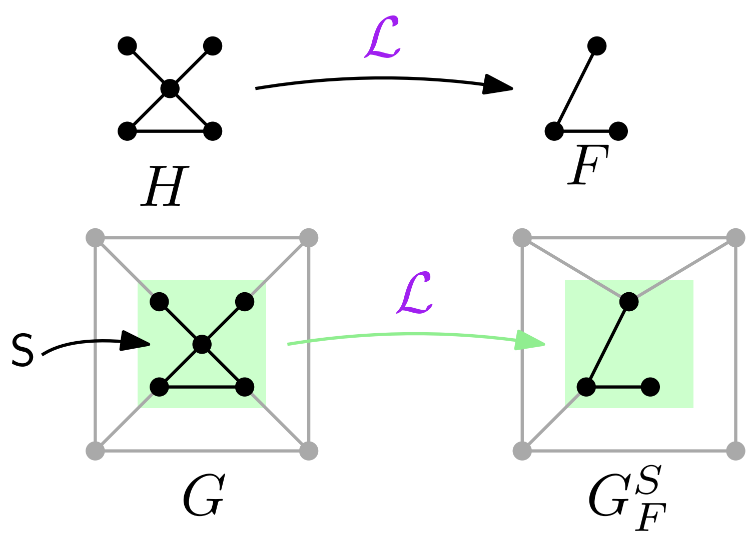

$\mathcal{L}$-Replacement to $\mathcal{H}$
Input: A graph G and an integer k.
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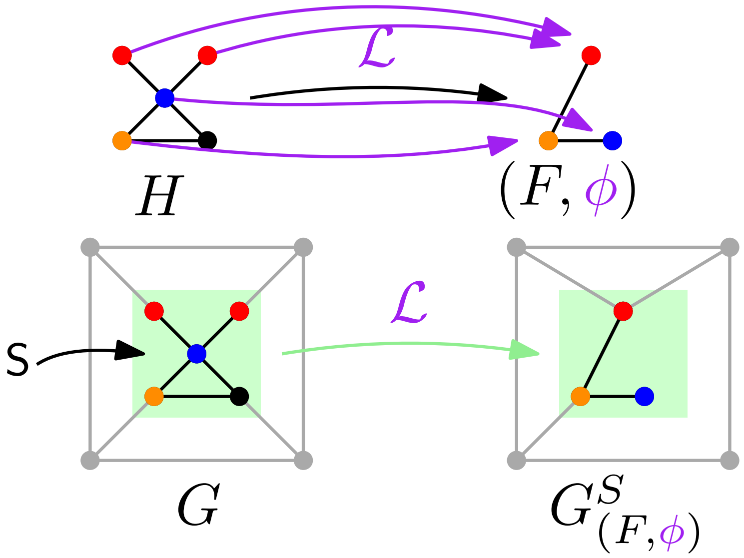

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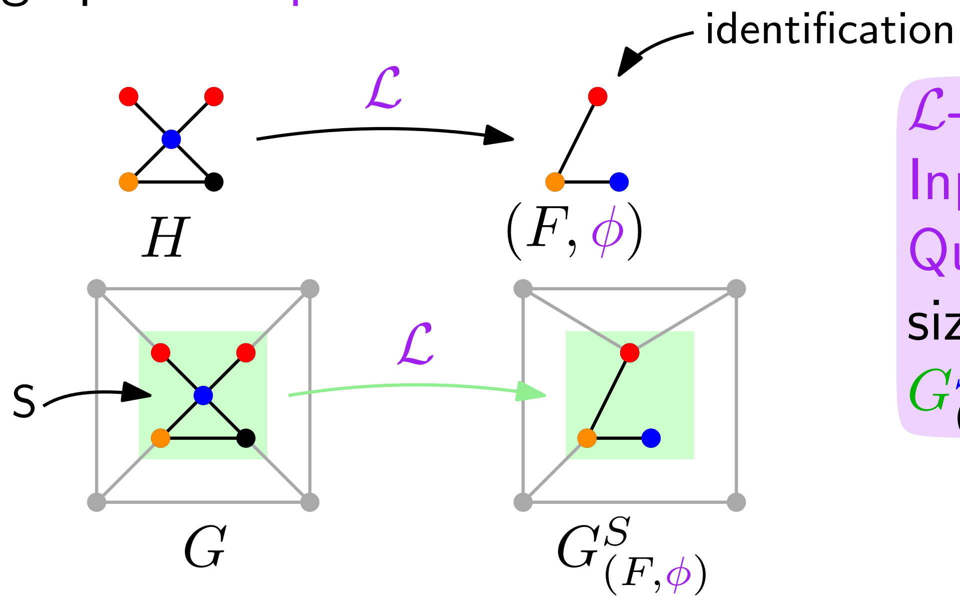

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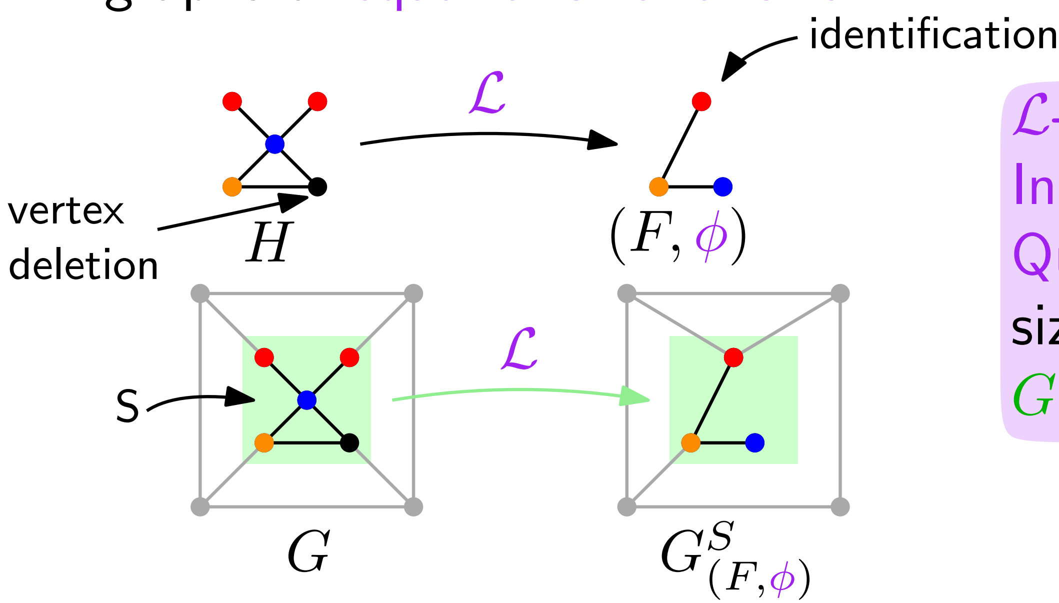

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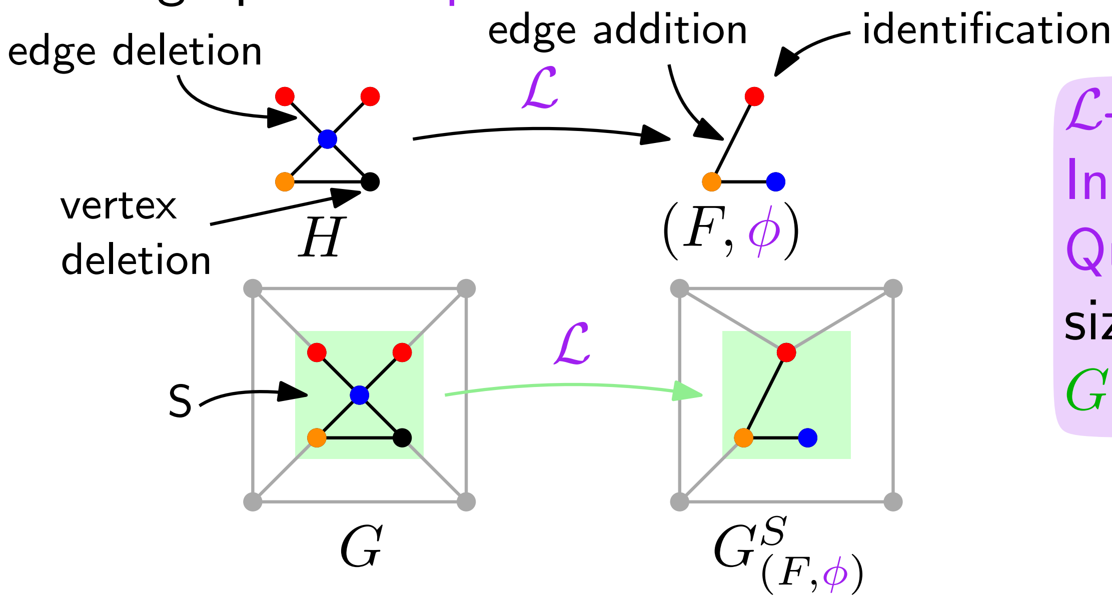

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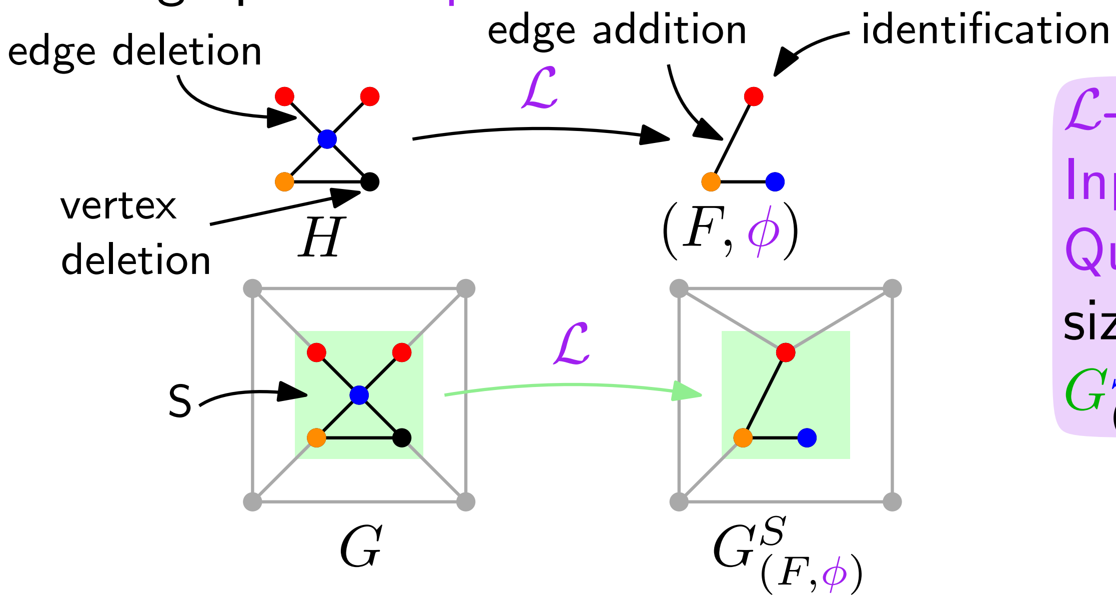

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$\mathcal{H}$ minor-closed

[Morelle, Sau, Thilikos]

$\mathcal{L}$-Replacement to $\mathcal{H}$ is solvable in time $2^{\mathrm{poly}_{\mathcal{H}}(k)} \cdot n^2$ for $\mathcal{L}$ hereditary.

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new dynamic programming

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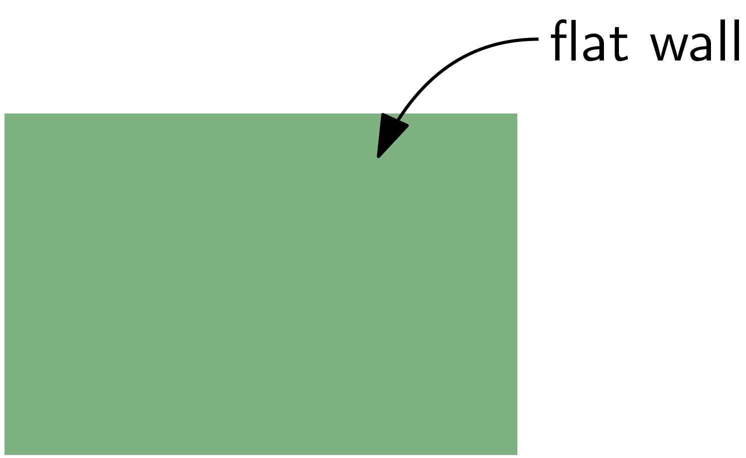
General case:



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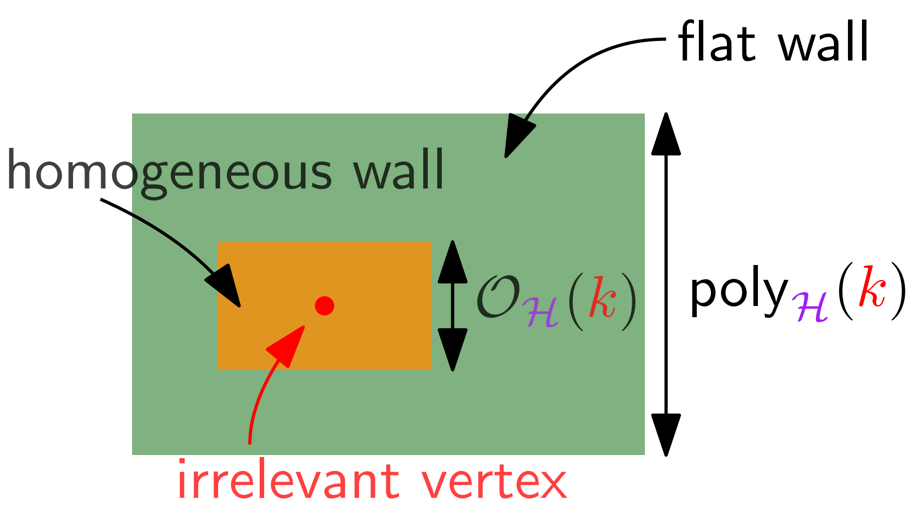
General case:



$\mathcal{H}$ minor-closed

[Morelle, Sau, Thilikos]

$\mathcal{L}$-Replacement to $\mathcal{H}$ is solvable in time $2^{\mathsf{poly}_{\mathcal{H}}(k)} \cdot n^2$ for $\mathcal{L}$ hereditary.

$\mathcal{H}_{\Sigma}$ = graphs embeddable on a surface Σ

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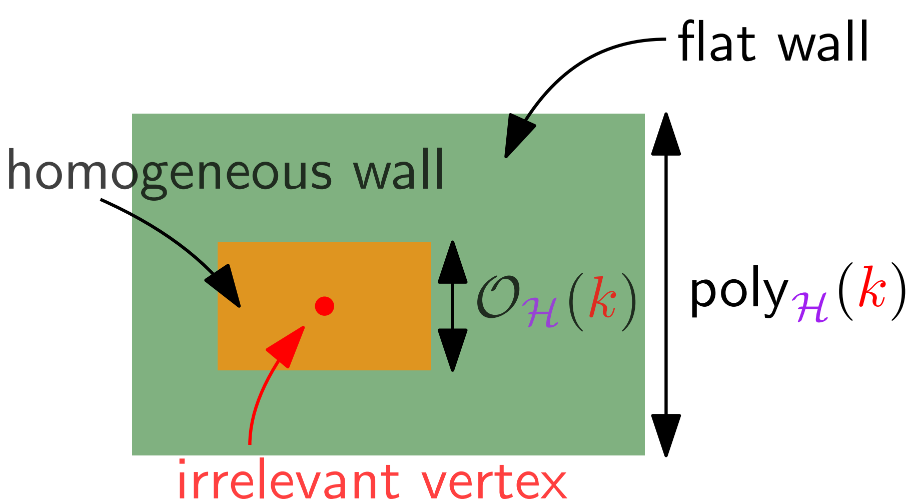
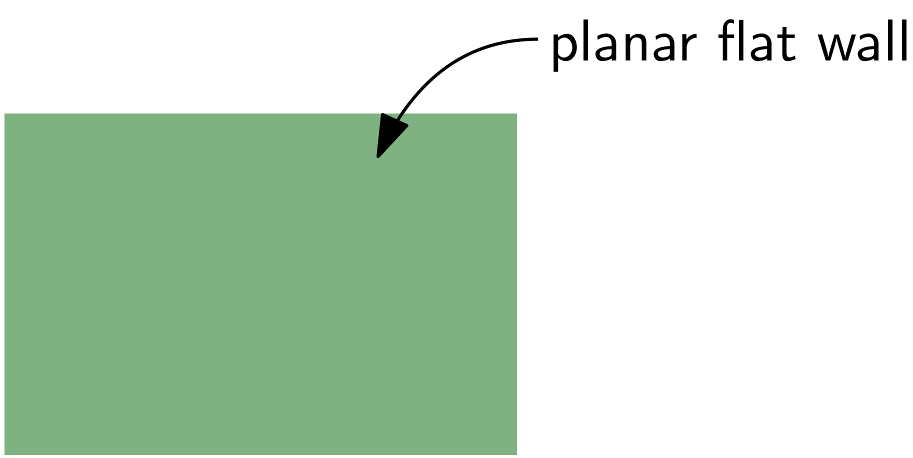


Case of surfaces:



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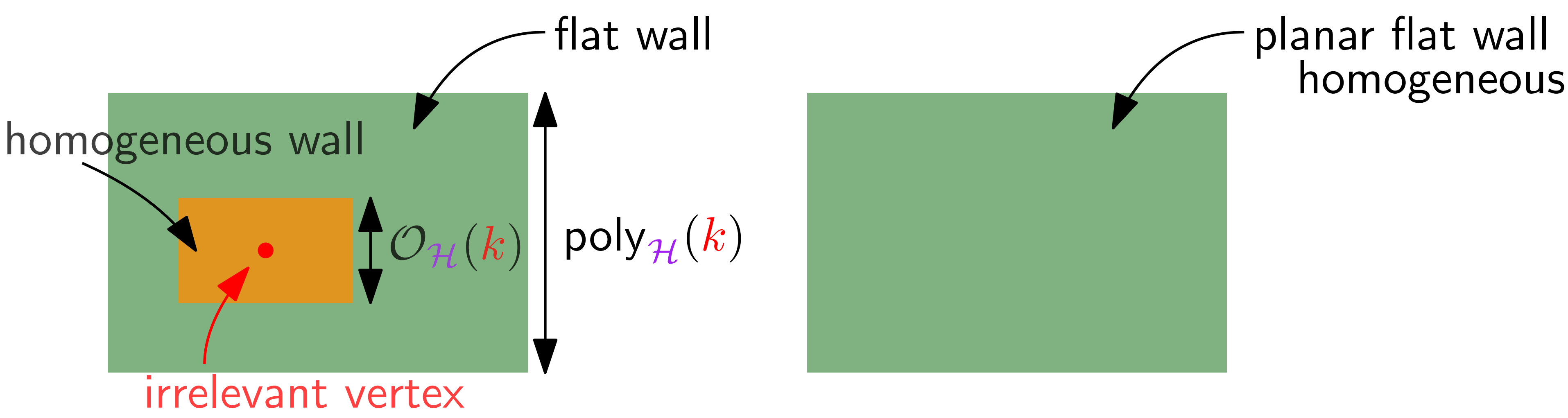

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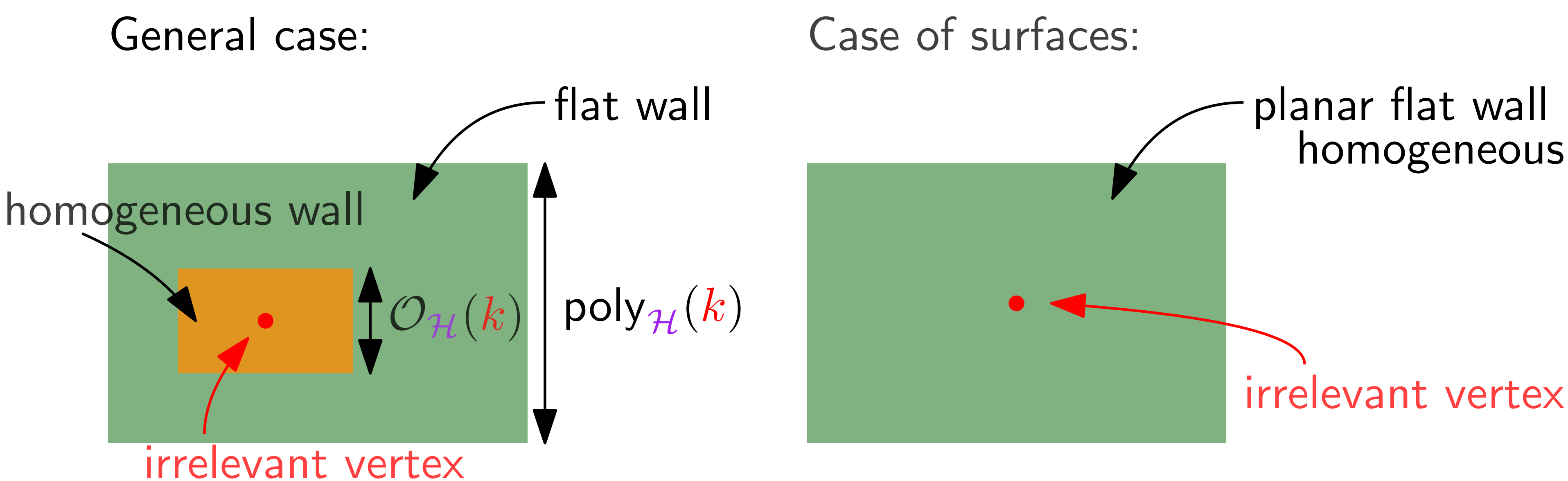
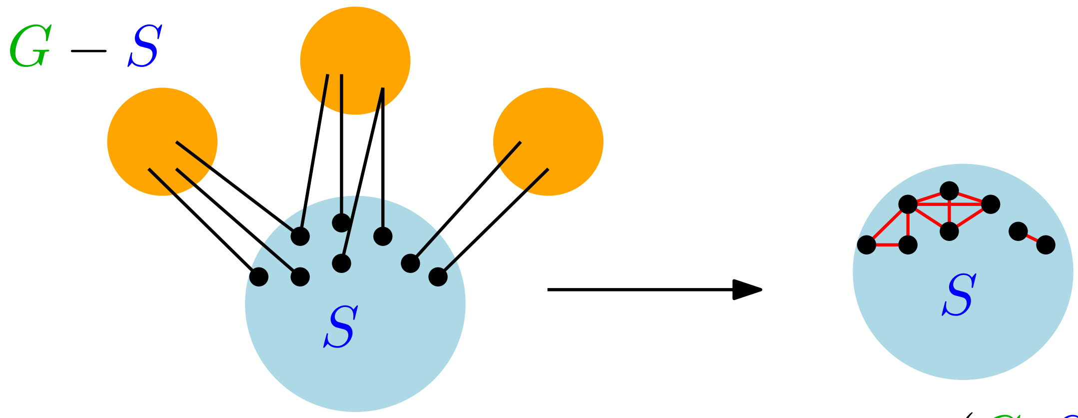


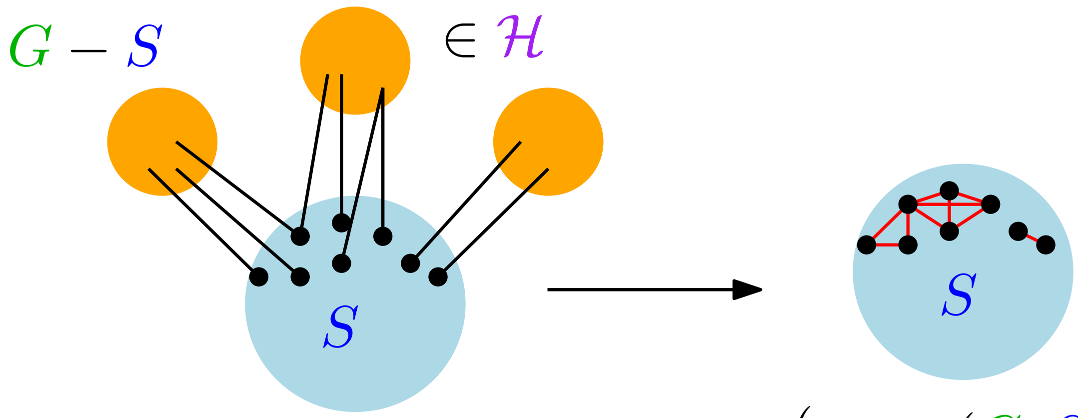
3. Measure p on the modulator

Torso of a vertex set S in a graph G:



torso(G, S)

Torso of a vertex set S in a graph G:

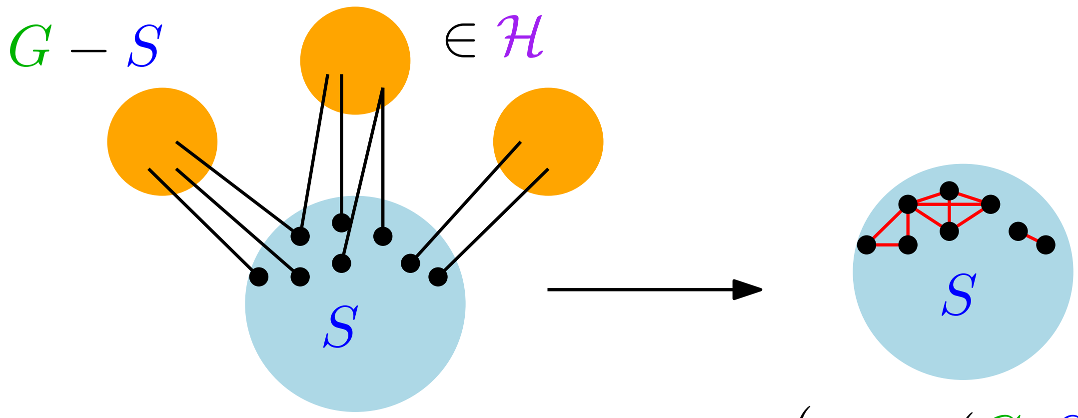


p(torso(G, S)) $\leq k$

Parameter p

$\mathcal{H}$-p$(G) = \min\{k \mid$ there is a vertex set S s.t. p(torso$(G, S)) \leq k$
and the components of $G - S$ are in $\mathcal{H}\}$

Torso of a vertex set S in a graph G:



$\text{p}(\ \text{torso}(G, S)\ ) \leq k$

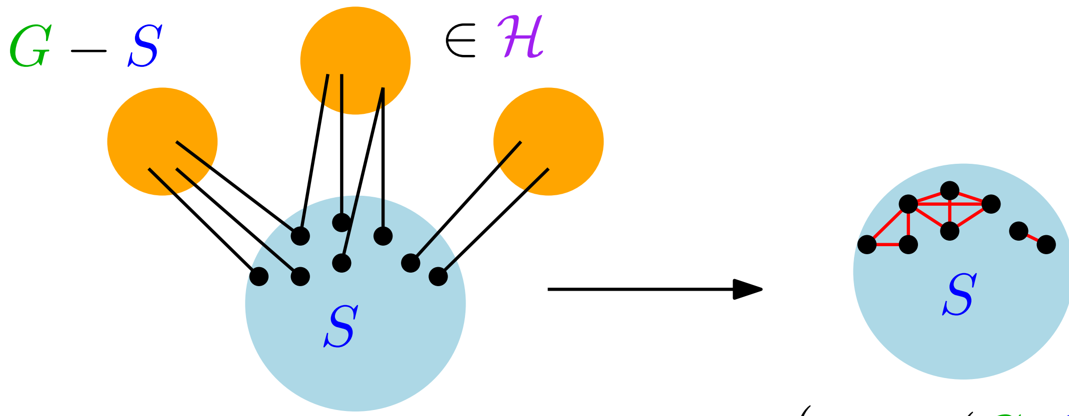
Parameter p

$\mathcal{H}\text{-p}(G) = \min\{k \mid \text{there is a vertex set } S \text{ s.t. } \text{p}(\text{torso}(G, S)) \leq k$
$\text{and the components of } G - S \text{ are in } \mathcal{H}\}$

Graph modification problem:

Input: A graph G and an integer k.
Output: Is $\mathcal{H}\text{-p}(G) \leq k$?

Torso of a vertex set S in a graph G:



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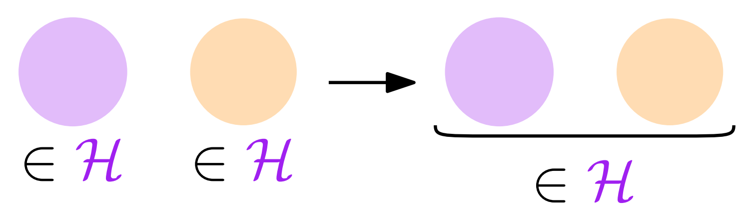
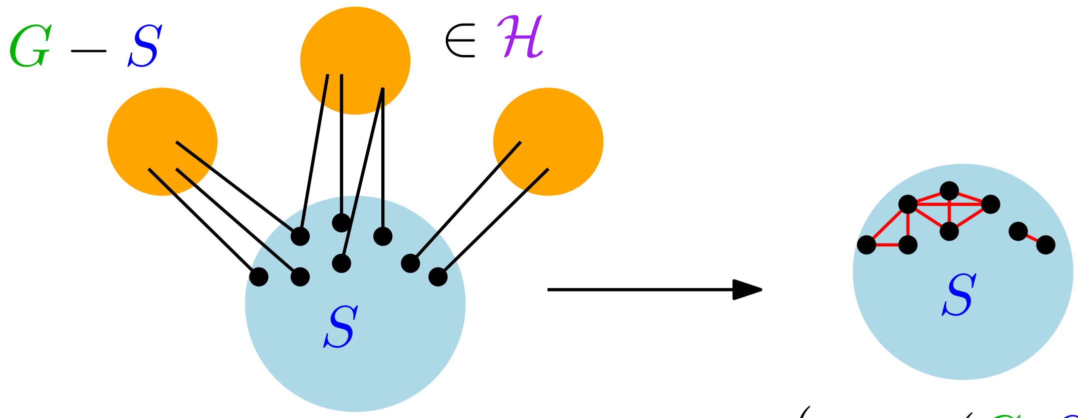
Input: A graph G and an integer k.
Output: Is $\mathcal{H}$-p$(G) \leq k$?

$\mathcal{H}$-size $\rightarrow$ Vertex Deletion to $\mathcal{H}$ if $\mathcal{H}$ is closed under disjoint union



Torso of a vertex set S in a graph G:



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Graph modification problem:

Input: A graph G and an integer k.
Output: Is $\mathcal{H}$-p$(G) \leq k$?

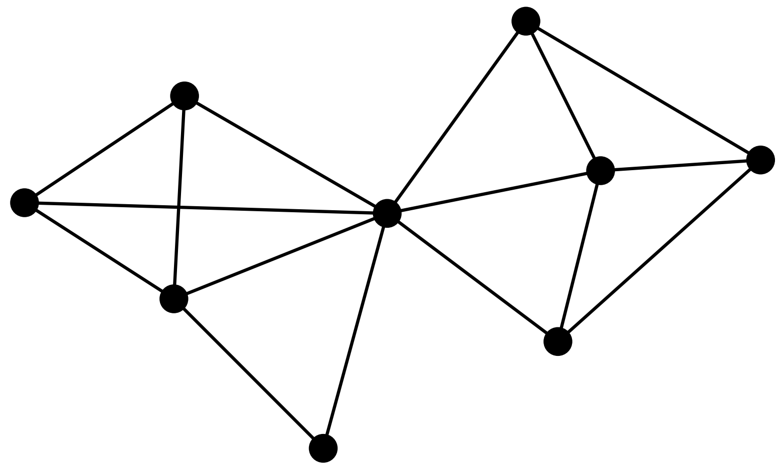
$\mathcal{H}$-size $\rightarrow$ Vertex Deletion to $\mathcal{H}$

$\mathcal{H}$-td $\rightarrow$ Elimination Distance to $\mathcal{H}$ [Bulian, Dawar, '16]

Torso of a vertex set S in a graph G:

treedepth td(G):

Step 0



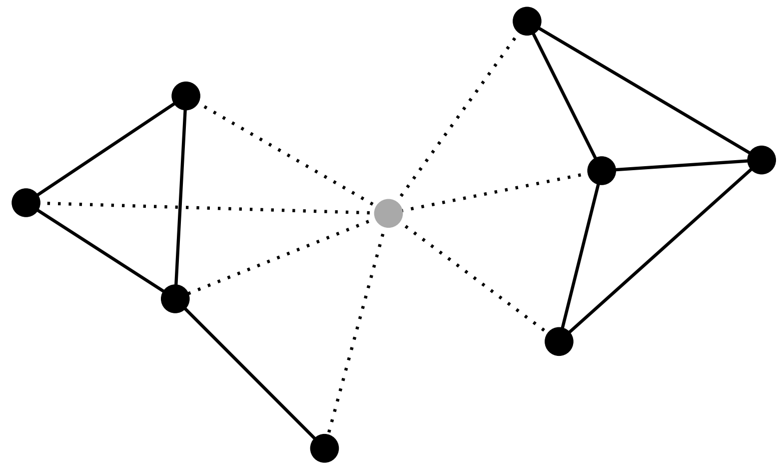
At each step, remove 1 vertex from each component

$\mathcal{H}$-td → ELIMINATION DISTANCE TO $\mathcal{H}$ [Bulian, Dawar, '16]

Torso of a vertex set S in a graph G:

treedepth td(G):

Step 1



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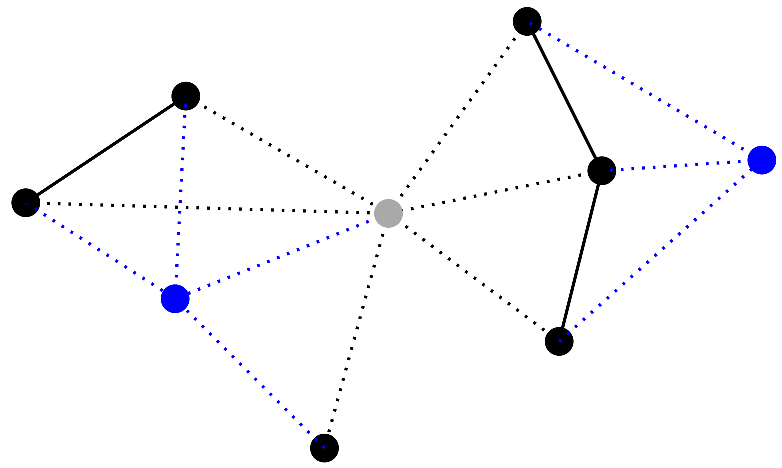
Torso of a vertex set S in a graph G:

treedepth td(G):

Step 2



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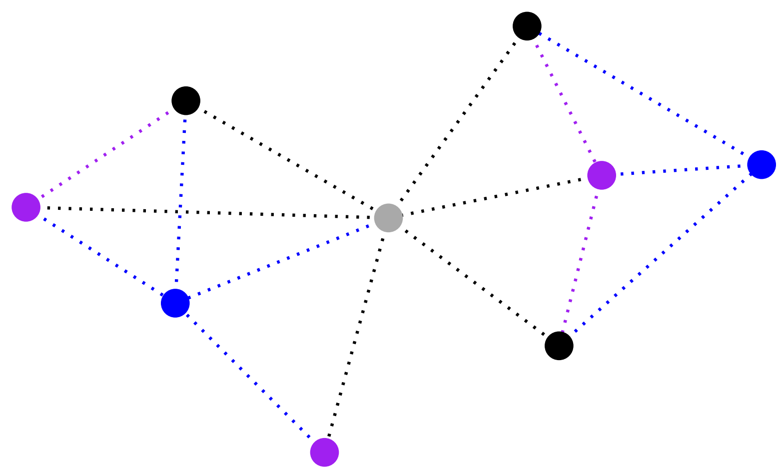
Torso of a vertex set S in a graph G:

treedepth td(G):

Step 3



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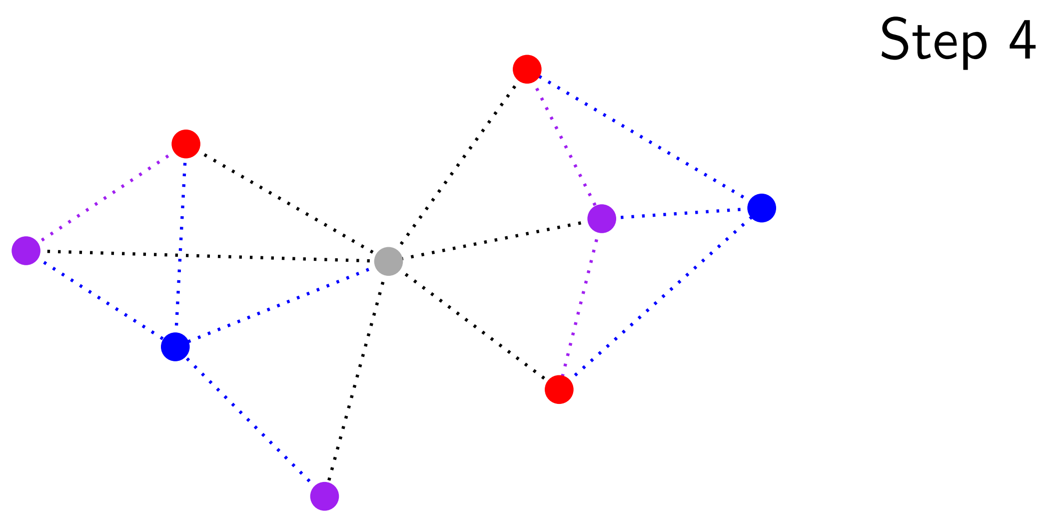
Torso of a vertex set S in a graph G:

treedepth td(G):



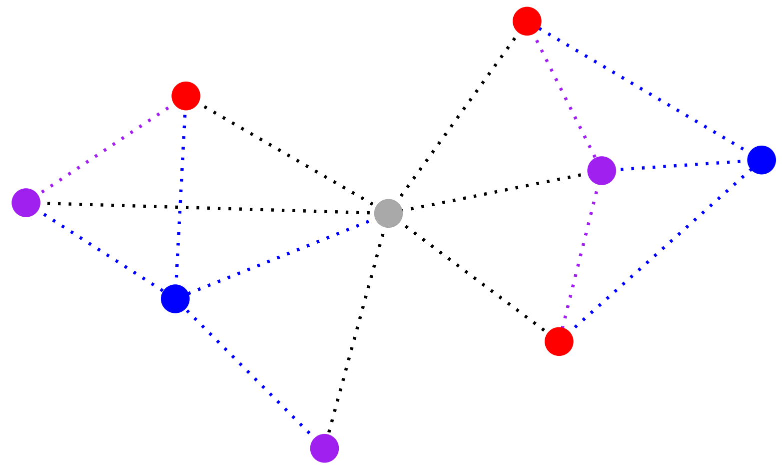
At each step, remove 1 vertex from each component

$\mathcal{H}$-td → ELIMINATION DISTANCE TO $\mathcal{H}$ [Bulian, Dawar, '16]

Torso of a vertex set S in a graph G:

treedepth td(G): min number of steps to remove all vertices

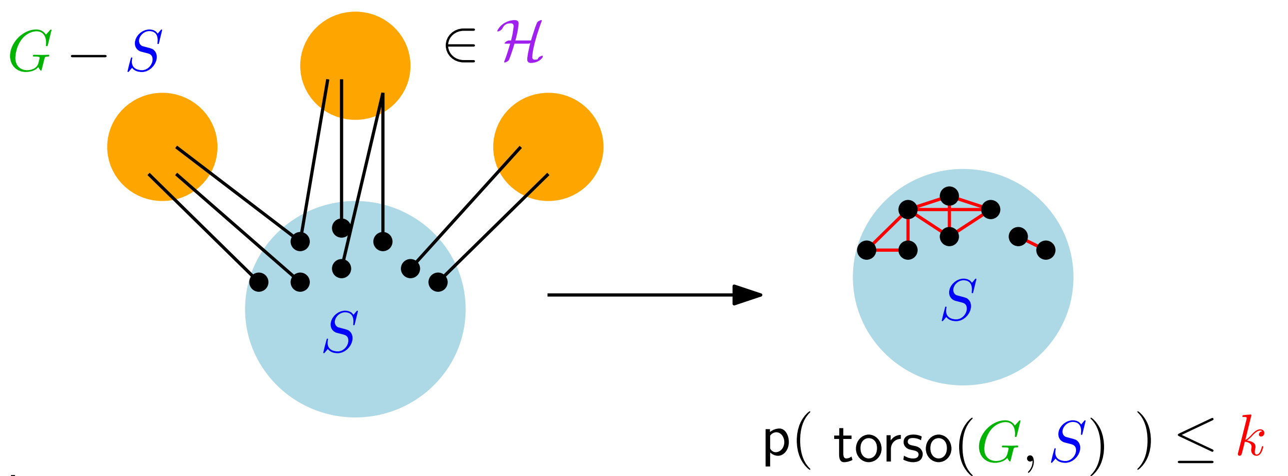
Step 4



At each step, remove 1 vertex from each component

$\mathcal{H}$-td → ELIMINATION DISTANCE TO $\mathcal{H}$ [Bulian, Dawar, '16]

Torso of a vertex set S in a graph G:



Parameter p

$\mathcal{H}$-p$(G) = \min\{k \mid$ there is a vertex set S s.t. p(torso$(G, S)) \leq k$
and the components of $G - S$ are in $\mathcal{H}\}$

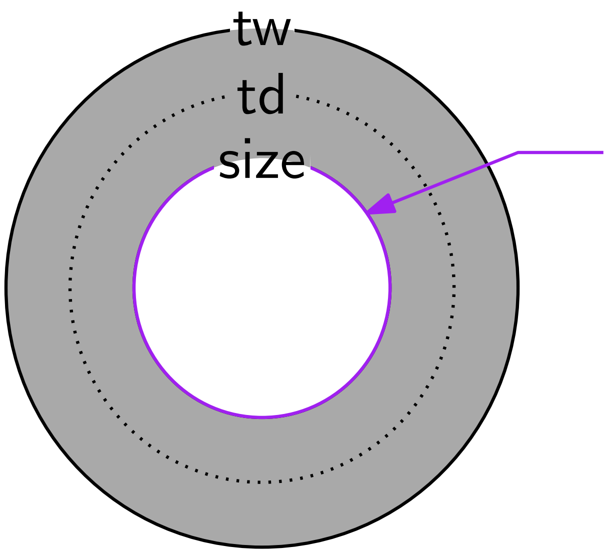
Graph modification problem:

Input: A graph G and an integer k.
Output: Is $\mathcal{H}$-p$(G) \leq k$?

$\mathcal{H}$-size $\rightarrow$ Vertex Deletion to $\mathcal{H}$

$\mathcal{H}$-td $\rightarrow$ Elimination Distance to $\mathcal{H}$ [Bulian, Dawar, '16]

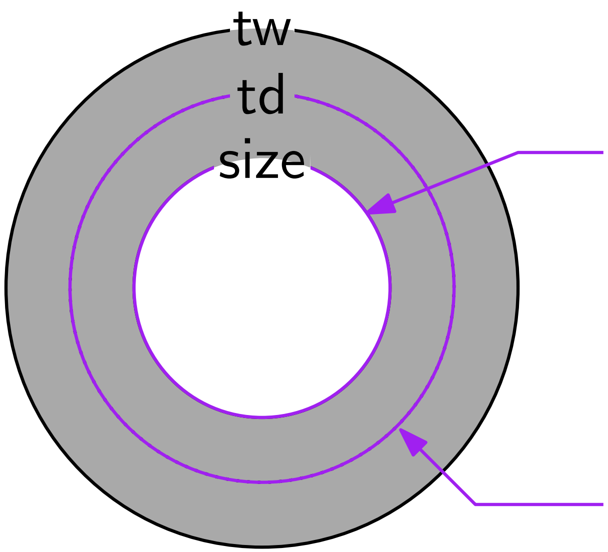
$\mathcal{H}$-tw$\rightarrow$ $\mathcal{H}$-Treewidth [Eiben, Ganian, Hamm, Kwon, '21] for each G, $\mathrm{tw}(G) \leq \mathrm{td}(G) \leq \mathrm{size}(G)$



Vertex Deletion to $\mathcal{H}$

[Morelle, Sau, Stamoulis, Thilikos]

$2^{\mathrm{poly}_{\mathcal{H}}(k)} \cdot n^2$ ← $\mathcal{H}$ minor-closed for each G, tw$(G) \leq$ td$(G) \leq$ size(G)



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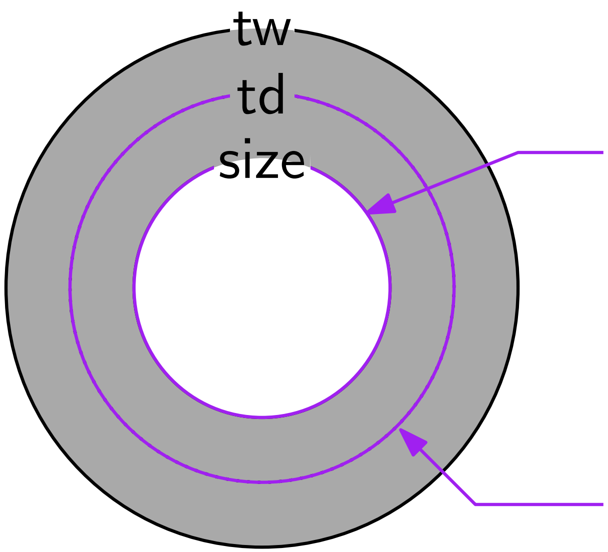
Elimination Distance to $\mathcal{H}$

[Robertson, Seymour, '04] + [Bulian, Dawar, '17] + [Kawarabayashi, Kobayashi, Reed, '12]

$f(k) \cdot n^2$ for some computable f

$\mathcal{H}$ minor-closed

for each G, $\mathsf{tw}(G) \leq \mathsf{td}(G) \leq \mathsf{size}(G)$



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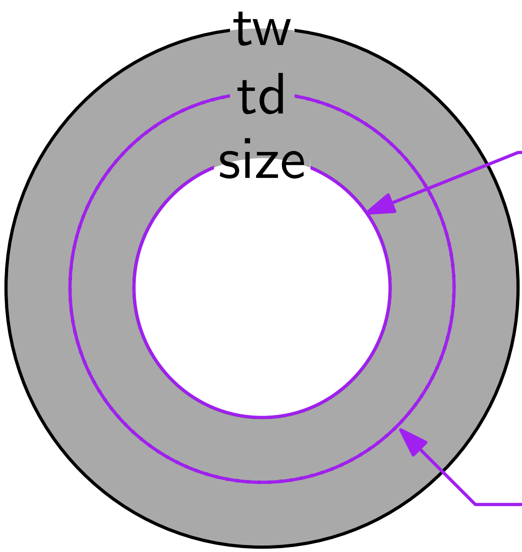
[Morelle, Sau, Stamoulis, Thilikos]

$2^{2^{2^{\mathsf{poly}_{\mathcal{H}}(k)}}} \cdot n^2$ ← $\mathcal{H}$ minor-closed for each G, $\mathrm{tw}(G) \leq \mathrm{td}(G) \leq \mathrm{size}(G)$



Vertex Deletion to $\mathcal{H}$

[Morelle, Sau, Stamoulis, Thilikos]

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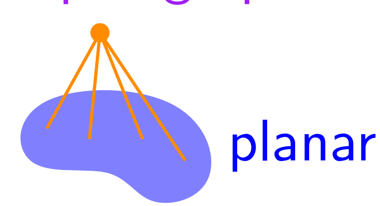
[Morelle, Sau, Stamoulis, Thilikos]

$2^{2^{2^{\mathrm{poly}_{\mathcal{H}}(k)}}} \cdot n^2$ ⟵ $\mathcal{H}$ minor-closed

$2^{\mathrm{poly}_{\mathcal{H}}(k)} \cdot n^3$ ⟵ $\mathcal{H}$ excludes an apex graph as a minor



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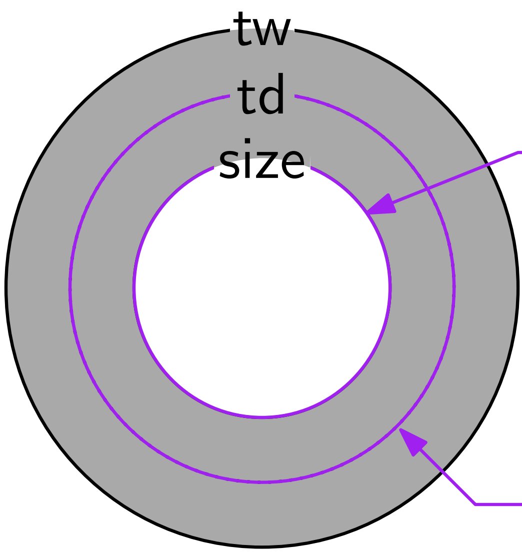

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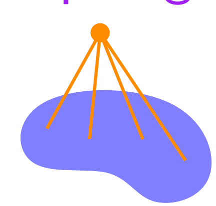

planar

Sketch of the proof for Vertex Deletion to $\mathcal{H}$:

Win/Win strategy on the **treewidth** tw(G) of G:

If G has big treewidth:

then G contains a big grid as a minor.

If there is a big flow from a set A to the grid:

then $A \cap S \neq \emptyset$ "obligatory set"

Branching step: guess $v \in A$ s.t. $v \in S$ and recurse on $(G - v, k - 1)$.

Otherwise there is a small flow to the grid:

then G contains a **"flat wall"**

Irrelevant vertex technique: there is a vertex v in the wall s.t. (G, k) and $(G - v, k)$ are equivalent instances. "irrelevant vertex"

Otherwise G has small treewidth:

then apply **dynamic programming** to conclude.

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Sketch of the proof for VERTEX DELETION TO $\mathcal{H}$:

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→ new dynamic programming $2^{\mathcal{O}(k \cdot \mathsf{tw} + \mathsf{tw} \log \mathsf{tw})} \cdot n$

Representative-based technique [Baste, Sau, Thilikos , '19]

DP for treedepth [Reidl, Rossmanith, Villaamil, Sikdar, '14]

Elimination Distance to $\mathcal{H}$:
Is there a vertex set S s.t. td(torso$(G, S)) \leq k$ and the components of $G - S$ are in $\mathcal{H}$?

Sketch of the proof

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$2^{2^{2^{\text{poly}_{\mathcal{H}}(k)}}} \cdot n^2$

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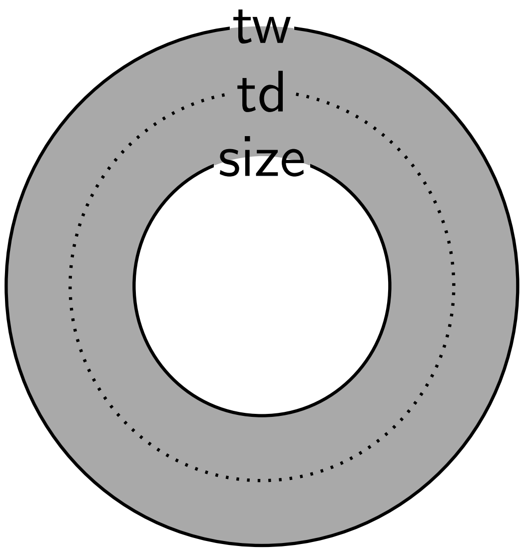
Limit of the irrelevant vertex technique



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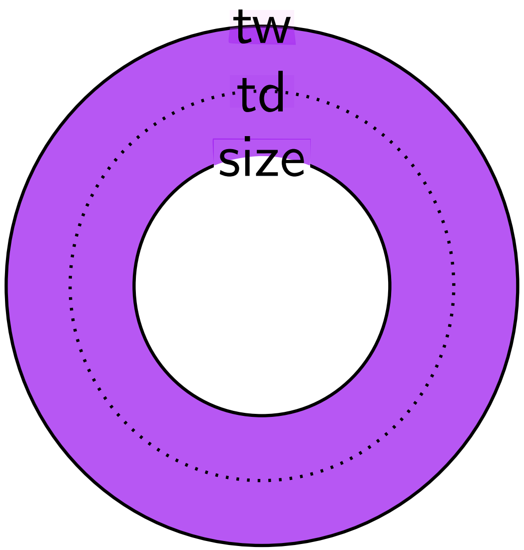
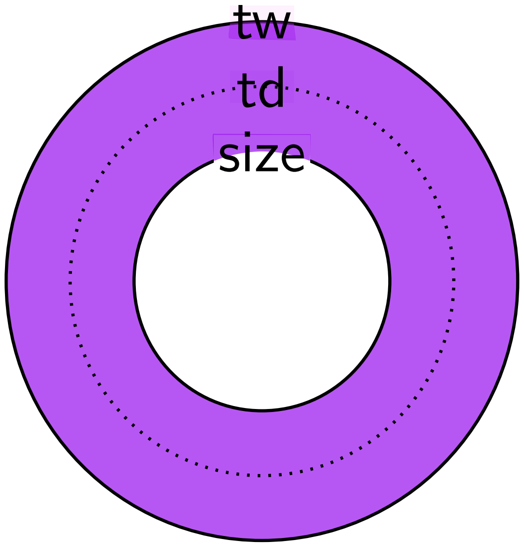


[Fomin, Golovach, Sau, Stamoulis, Thilikos, '23]

For $\mathcal{H}$ minor-closed, the irrelevant vertex technique works for any parameter $\mathcal{H}$-p such that size $\geq$ p $\geq$ tw.

Limit of the irrelevant vertex technique



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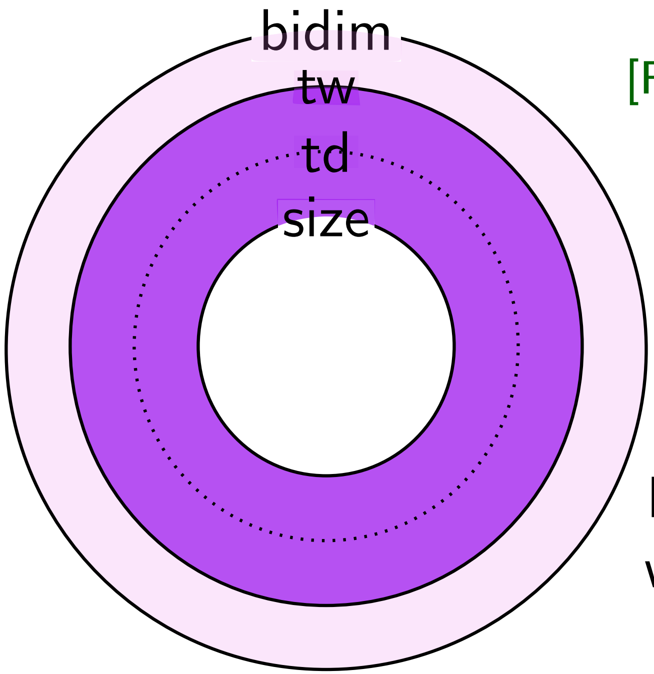
For $\mathcal{H}$ minor-closed, the irrelevant vertex technique works for any parameter $\mathcal{H}$-p such that size $\geq$ p $\geq$ tw. "up to modulators of bounded tw"

Limit of the irrelevant vertex technique



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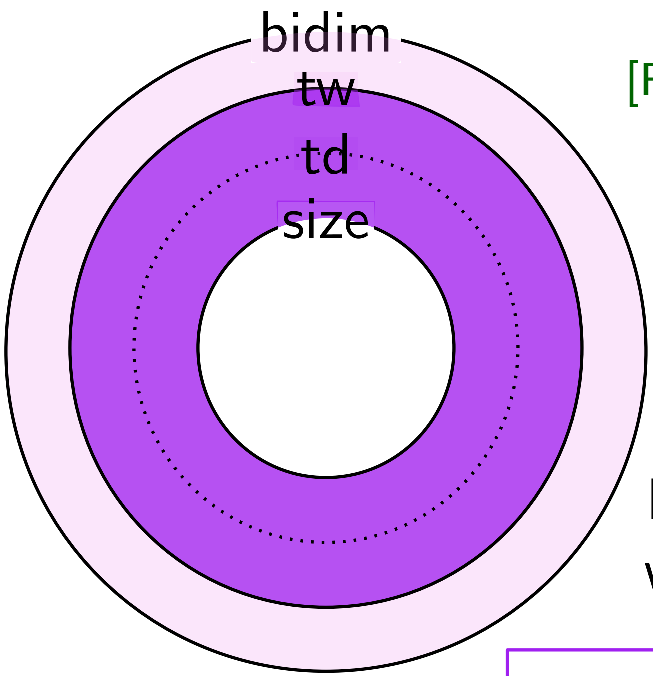
[Sau, Stamoulis, Thilikos, '25]

For $\mathcal{H}$ minor-closed, the irrelevant vertex technique works up to modulators of bounded **bidimensionality**.

Limit of the irrelevant vertex technique



[Fomin, Golovach, Sau, Stamoulis, Thilikos, '23]

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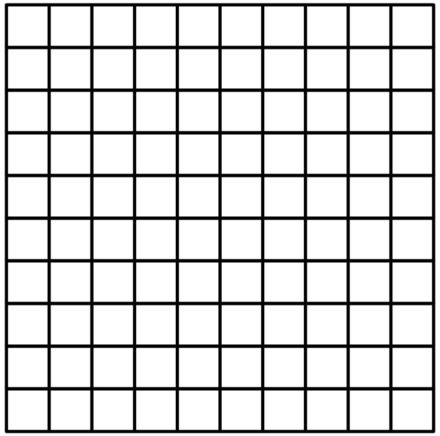
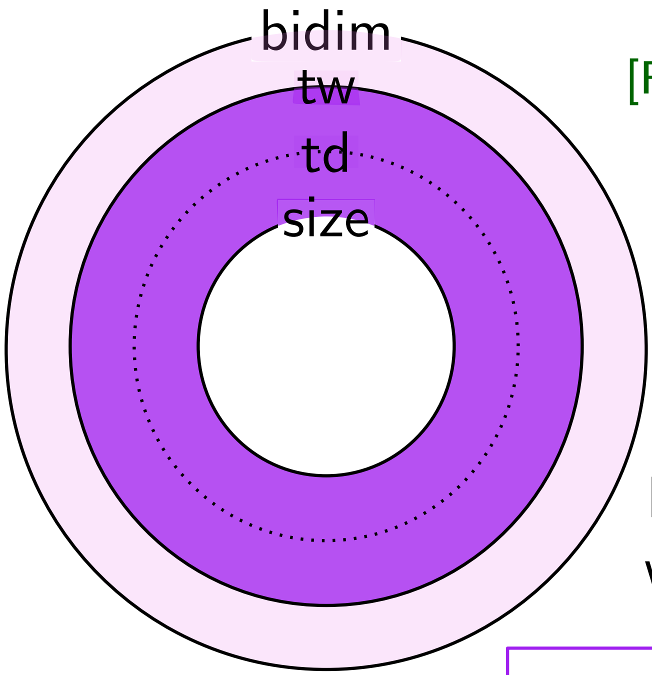
[Sau, Stamoulis, Thilikos, '25]

For $\mathcal{H}$ minor-closed, the irrelevant vertex technique works up to modulators of bounded **bidimensionality**.

bidim$(G, S) =$ max treewidth of an S-minor of G.



Limit of the irrelevant vertex technique



[Fomin, Golovach, Sau, Stamoulis, Thilikos, '23]

For $\mathcal{H}$ minor-closed, the irrelevant vertex technique works for any parameter $\mathcal{H}$-p such that size $\geq$ p $\geq$ tw.

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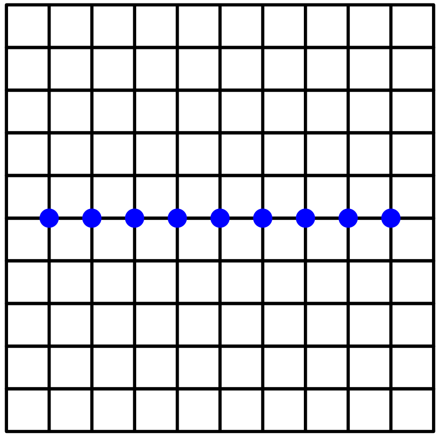

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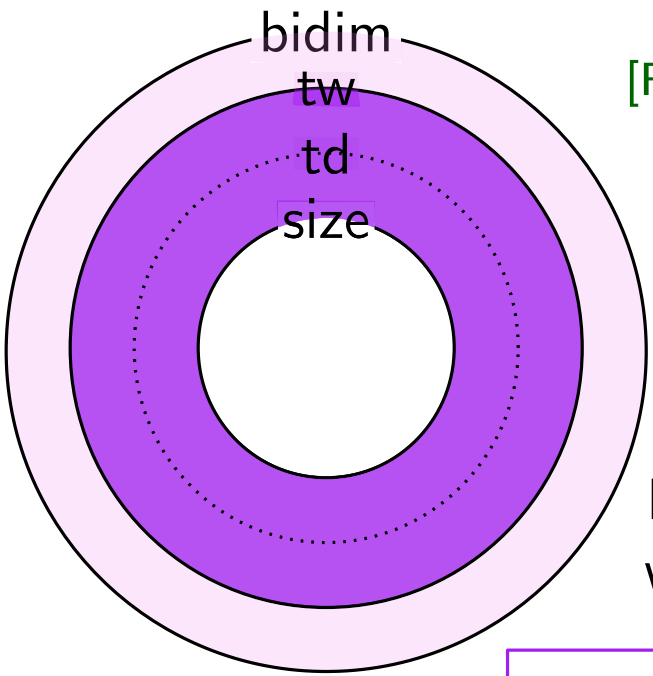

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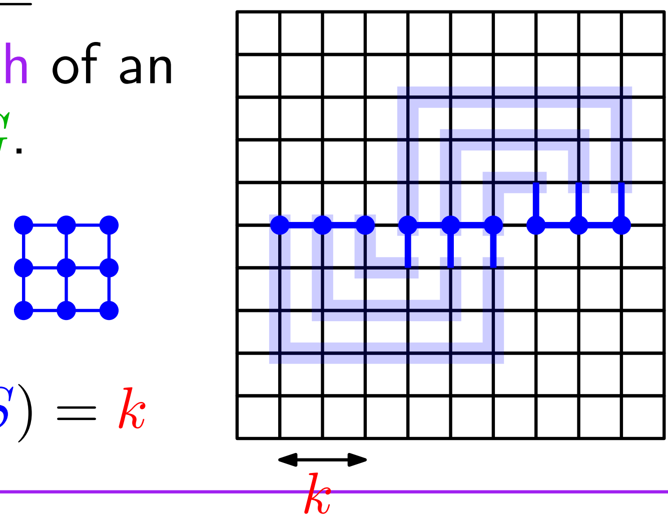

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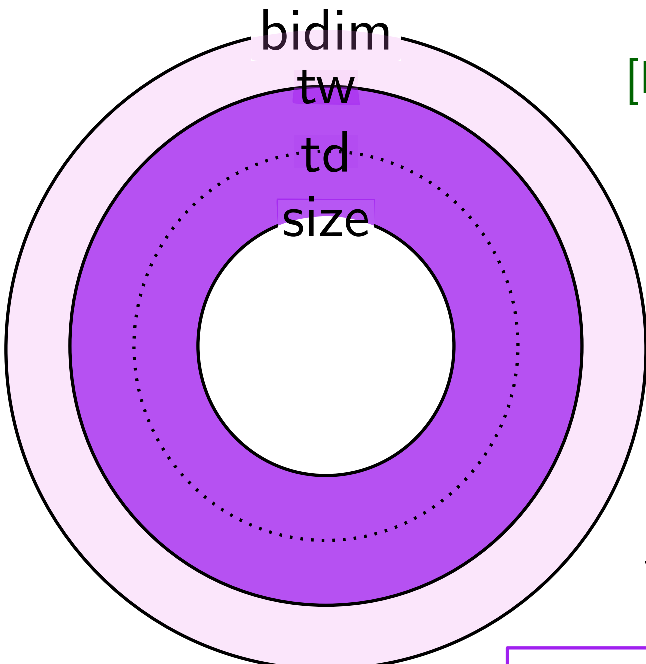

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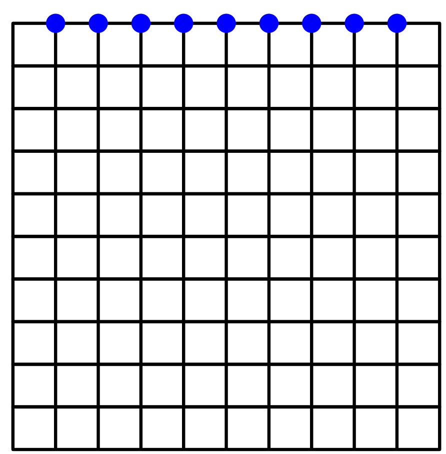

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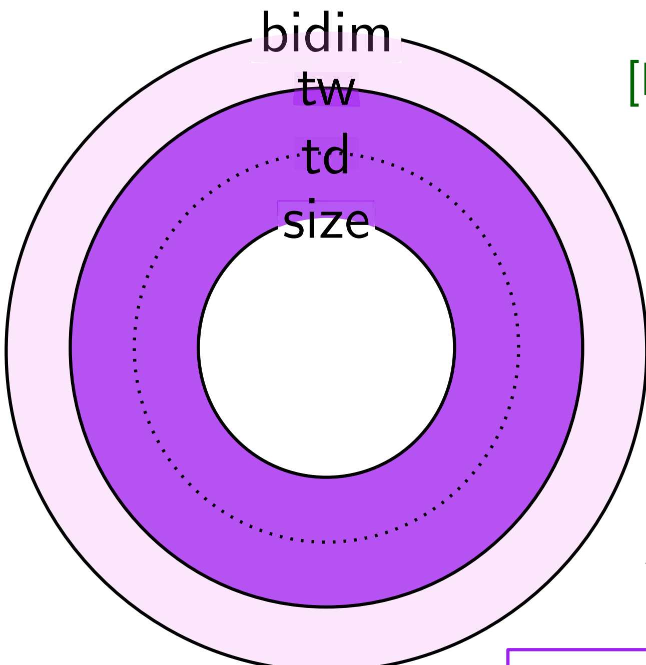

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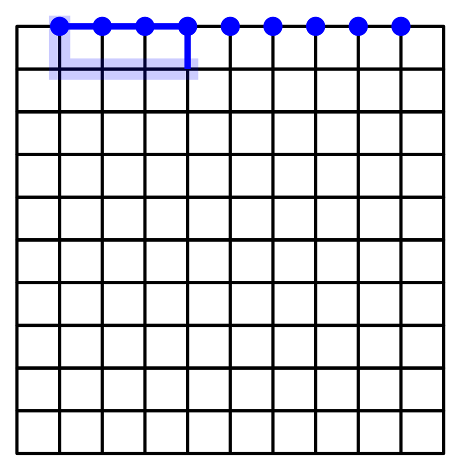

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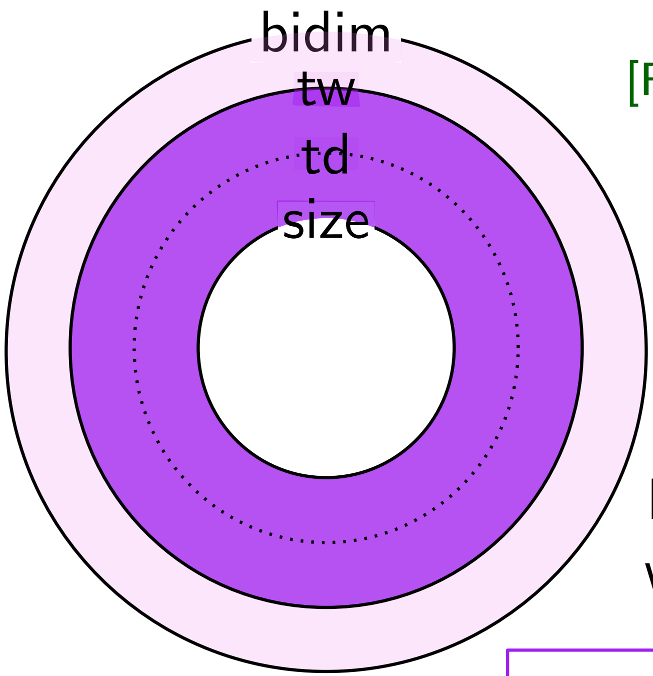

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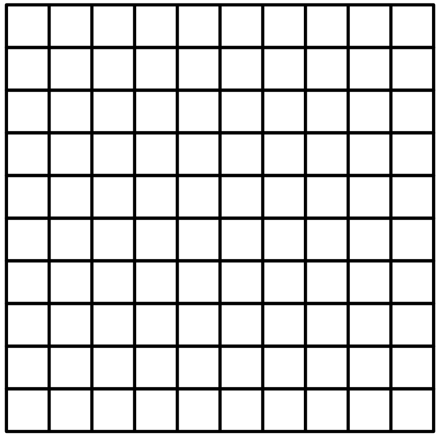
"max size of a grid grasped by S"



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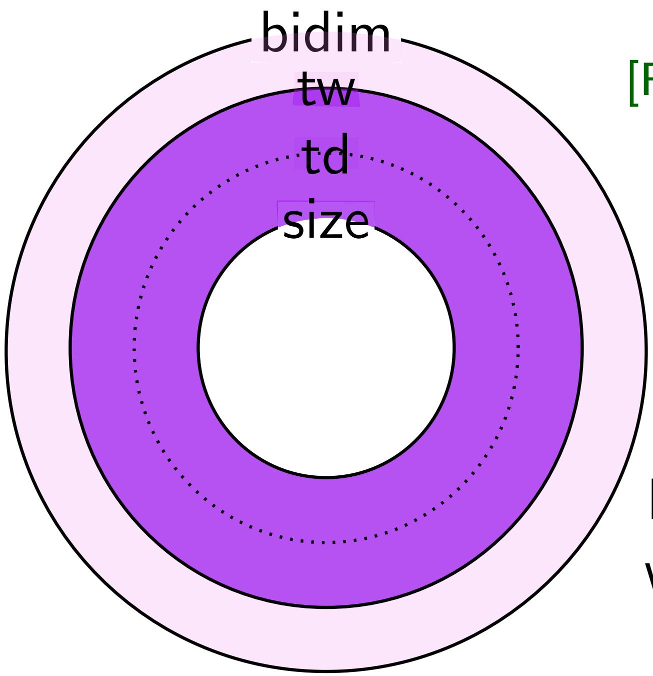

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Any **graph modification problem** where:

- the **modulator** has bounded bidimensionality
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can be solved in time $f(k) \cdot n^2$, for some computable f.

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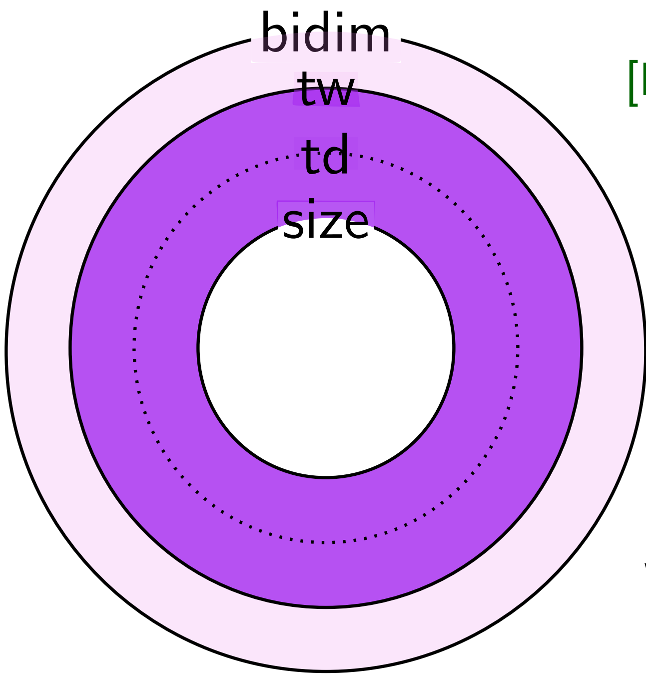

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variables v, e, V, E
quantifiers $\forall$, $\exists$
connectives $\wedge$, $\vee$, $\Rightarrow$, $\neg$, $\in$
relations inc$(\cdot)$, $|\cdot| = q \mod r$

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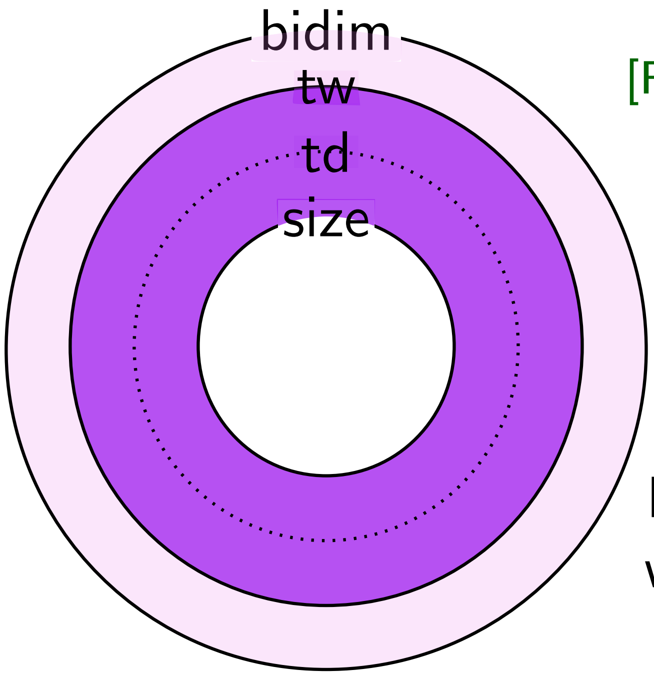

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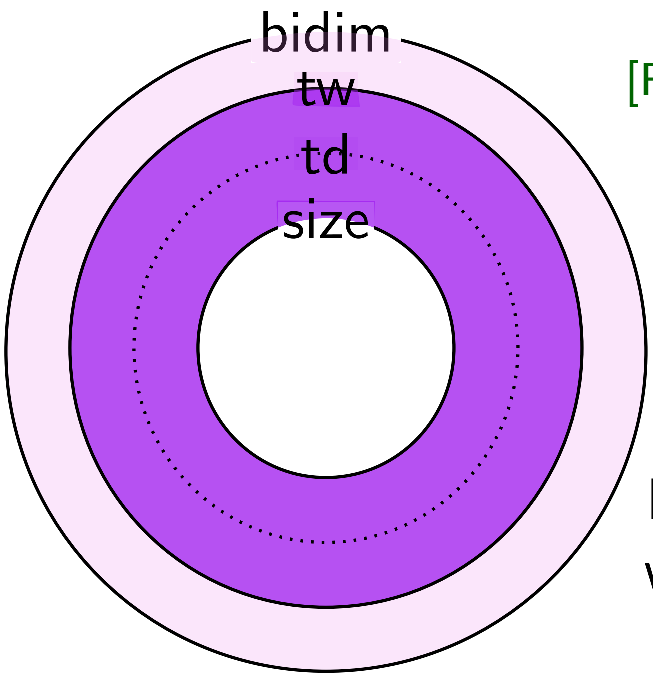

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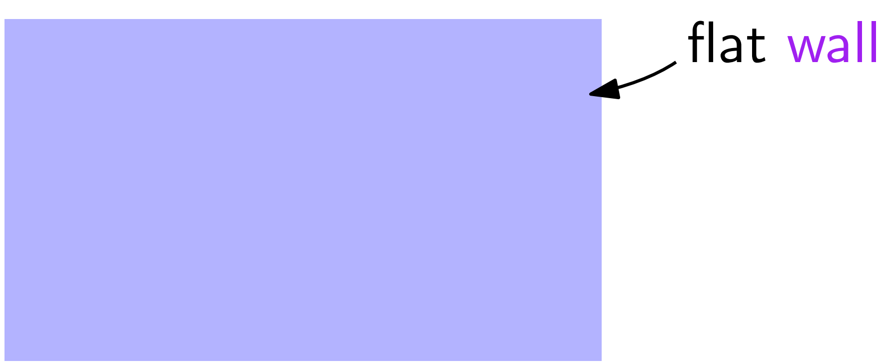

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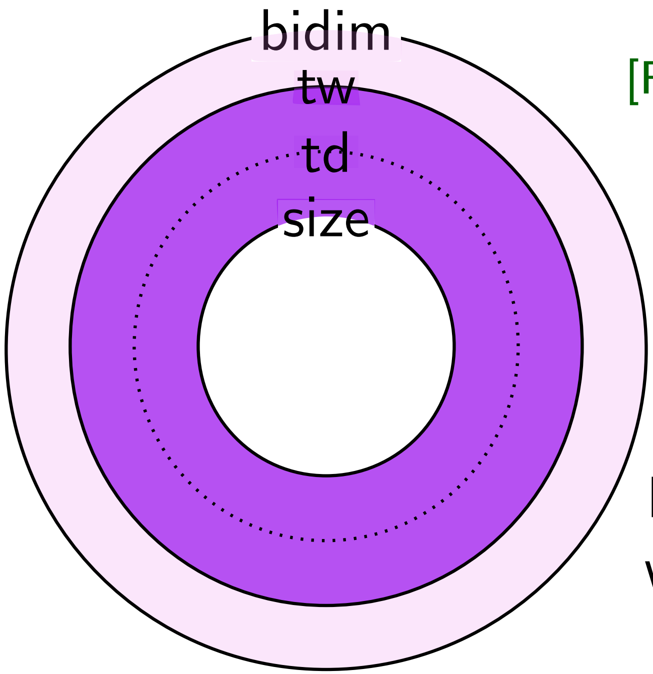

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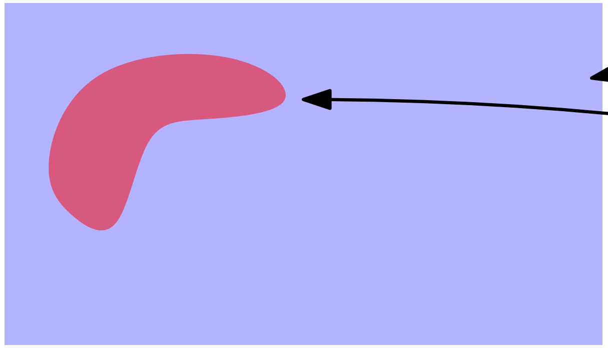
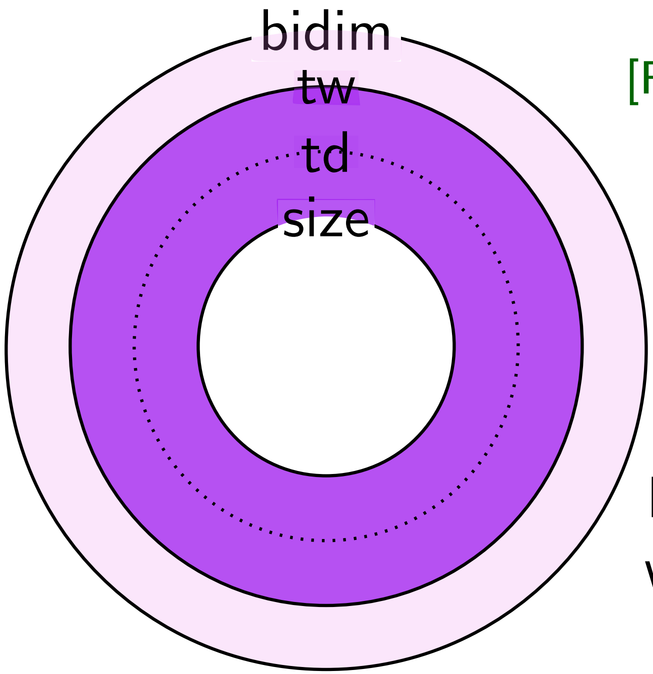


flat wall

no matter how we delete/modify the modulator

Limit of the irrelevant vertex technique



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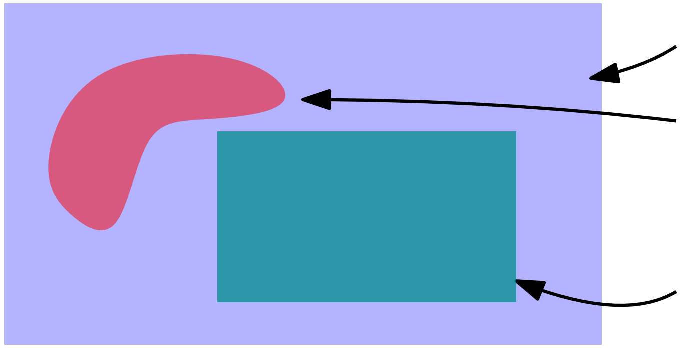
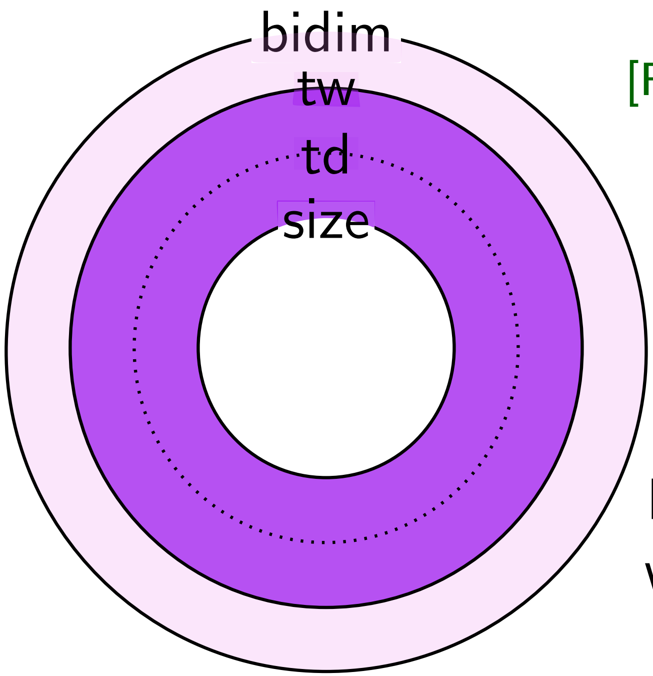


- flat wall
- no matter how we delete/modify the modulator
- there is a big enough subwall that is not modified.

Limit of the irrelevant vertex technique



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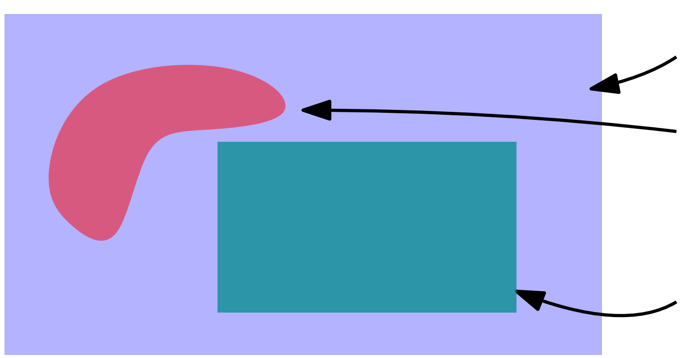

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Breaking the limit

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How to solve a graph modification problem where the modulator has unbounded bidimensionality?

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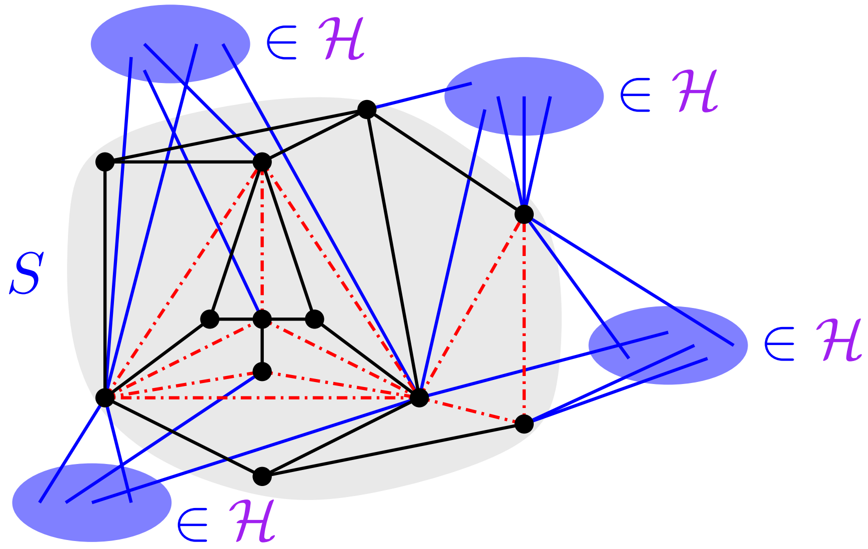
$\mathcal{H}$-PLANARITY
Input: A graph G.
Output: Is there a vertex set S whose torso is **planar** and s.t. the connected components of $G - S$ are in $\mathcal{H}$?



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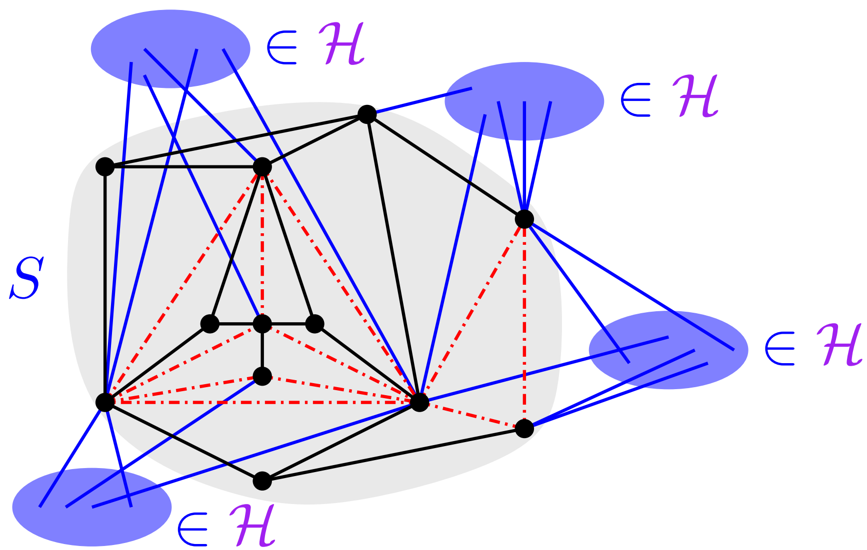

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If $\mathcal{H}$ is hereditary, CMSO-definable, and decidable in time $\mathcal{O}(n^c)$, then $\mathcal{H}$-PLANARITY is solvable in time $\mathcal{O}(n^4 + n^c \log n)$.

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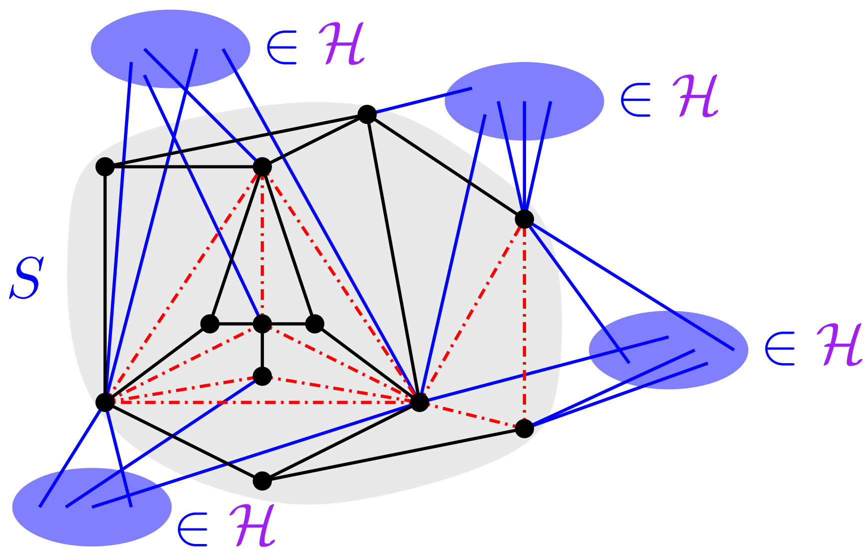

if $G \in \mathcal{H}$, then $G - v \in \mathcal{H}$

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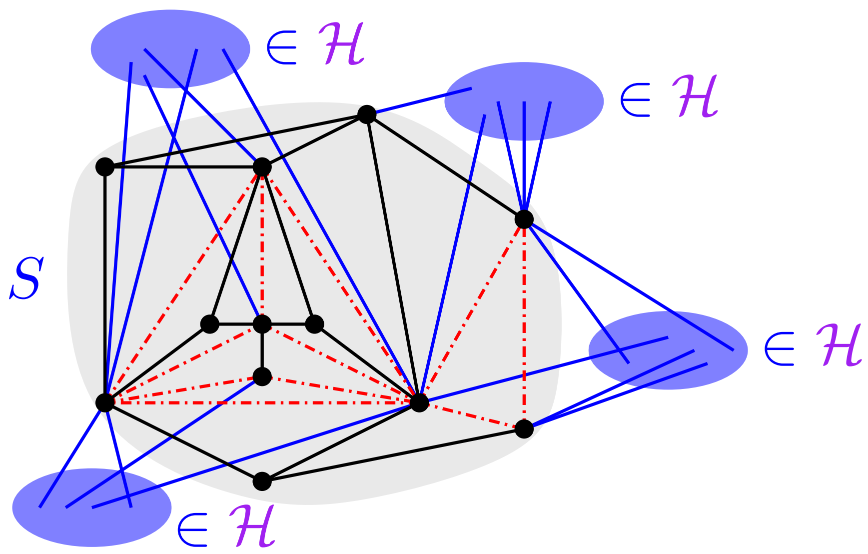

more general than minor-closed

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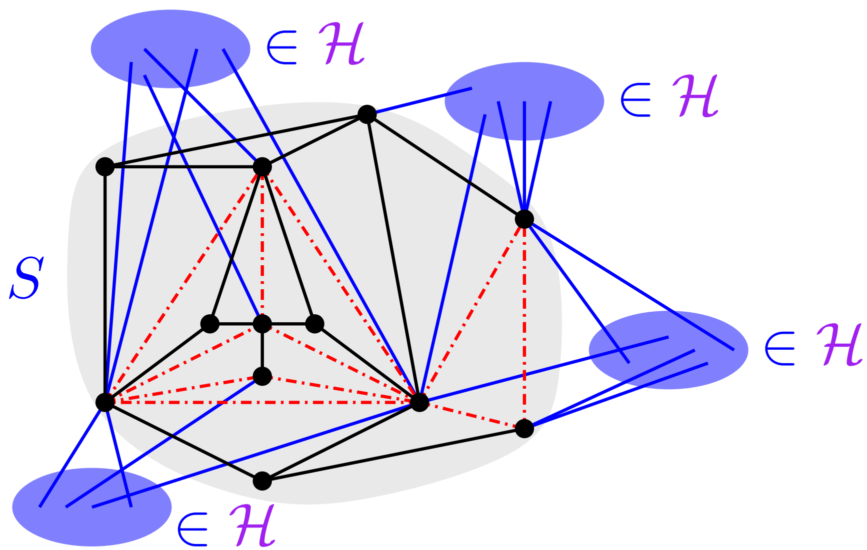

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$\rightarrow$ new irrelevant vertex technique

Sketch of the proof

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[Lokshtanov, Ramanujan, Saurabh, Zehavi, '18]

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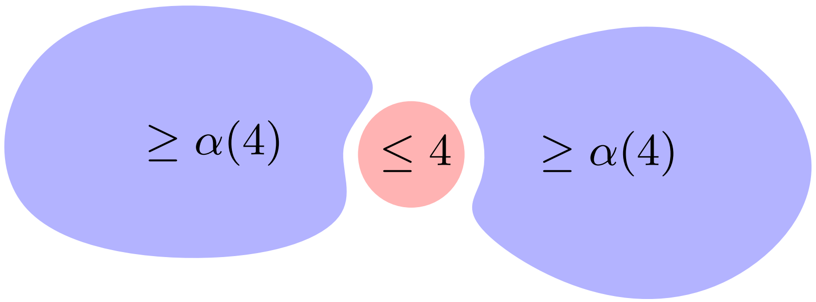

$(\alpha(4), 4)$-breakable graph

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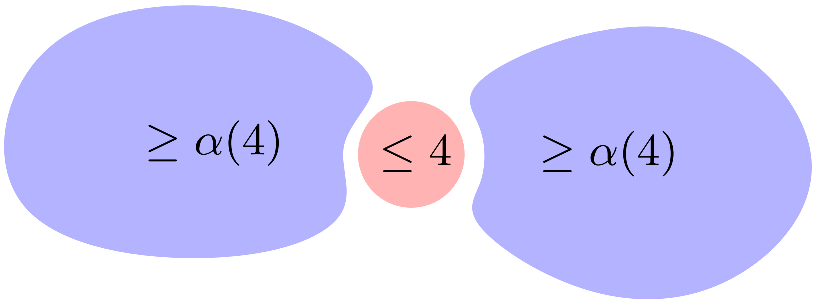

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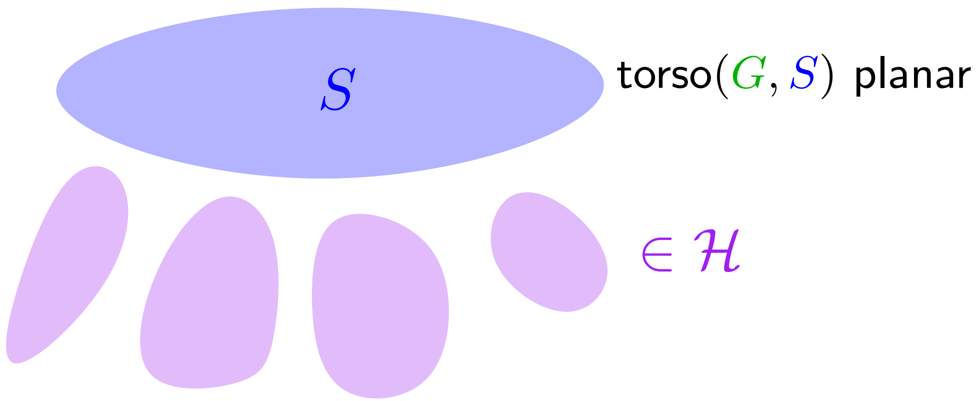
$\mathcal{H}$-**planar** graph



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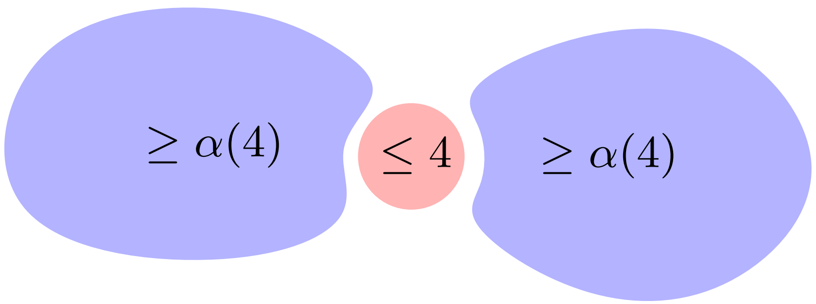

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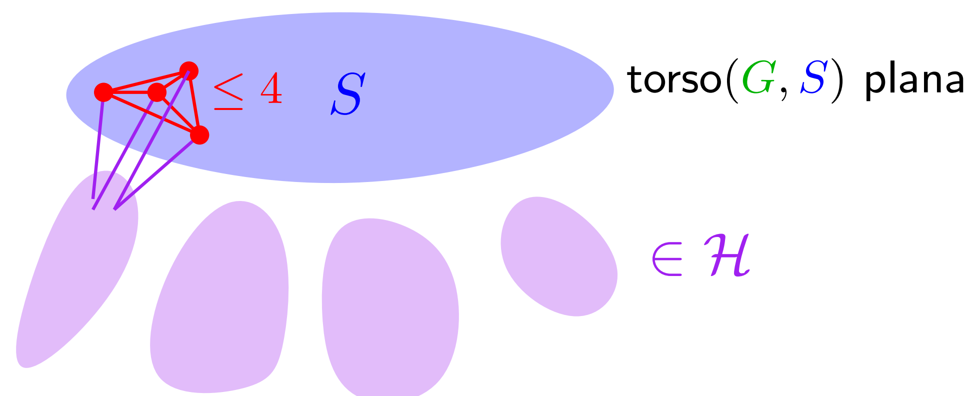

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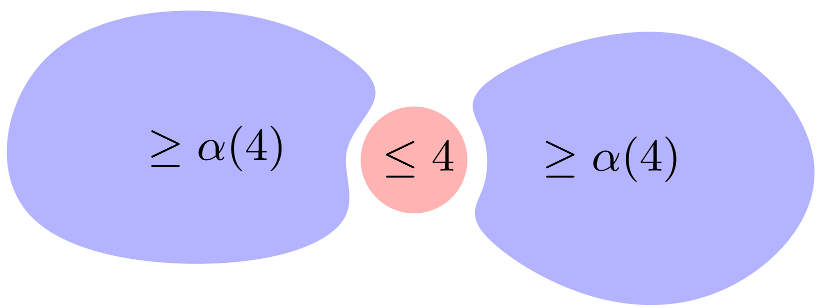

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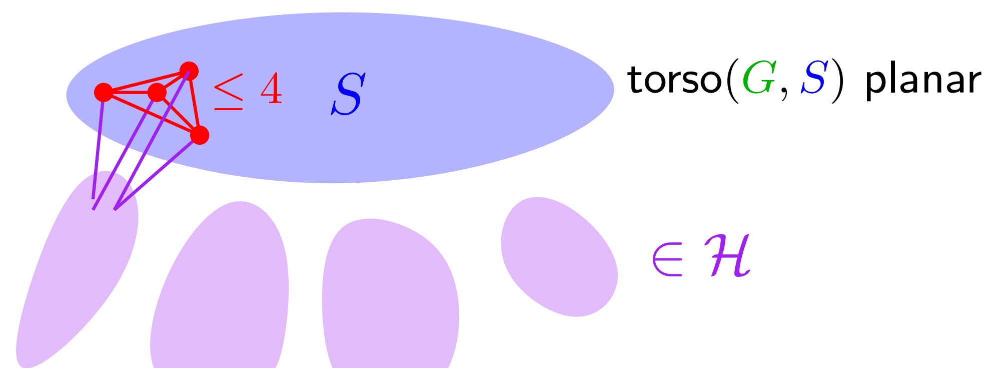

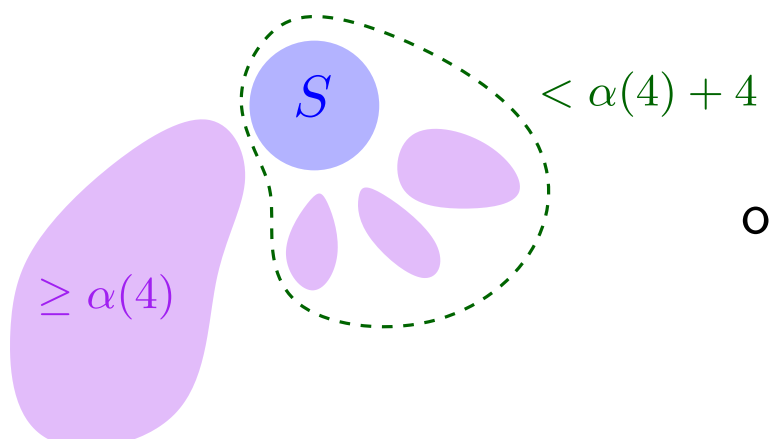

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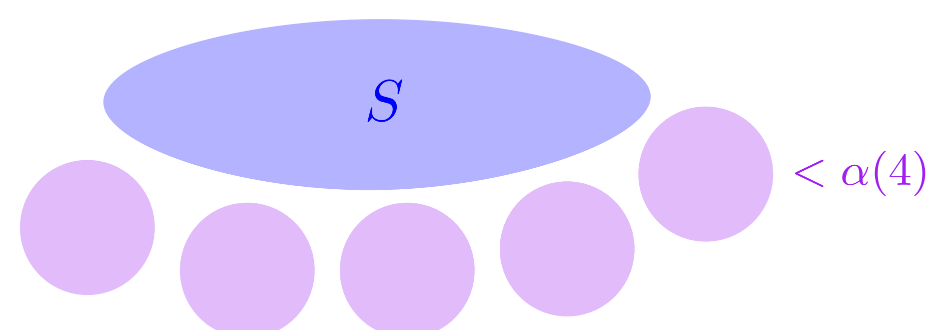

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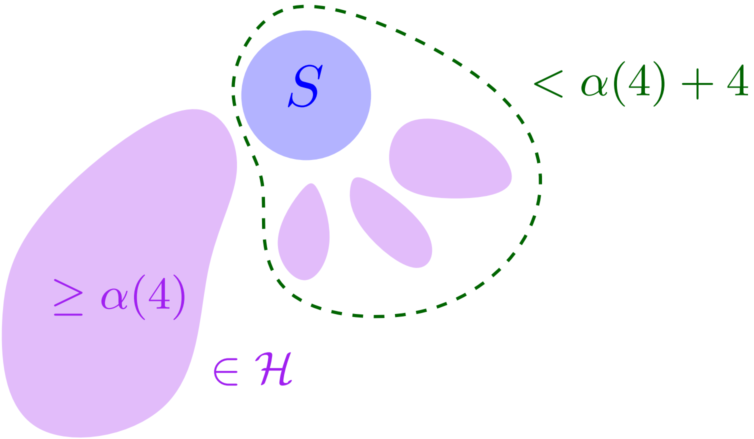

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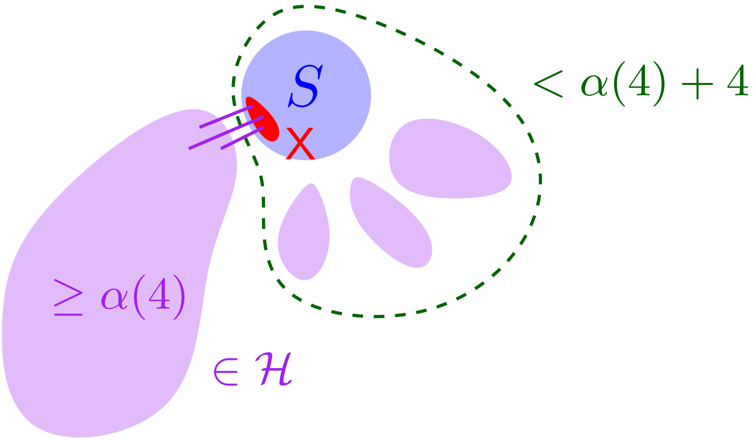

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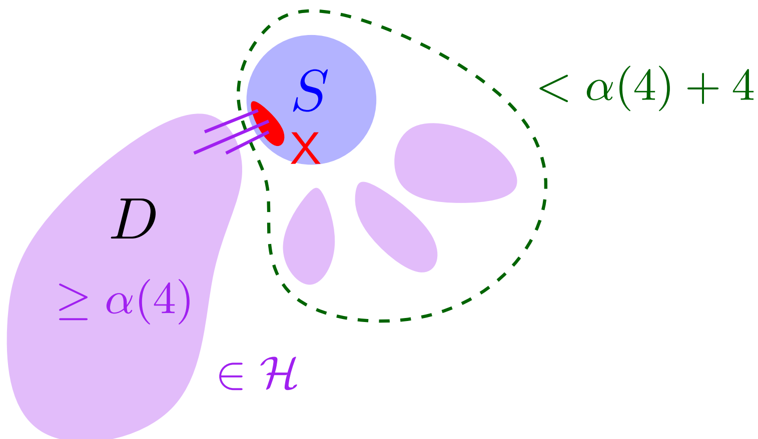

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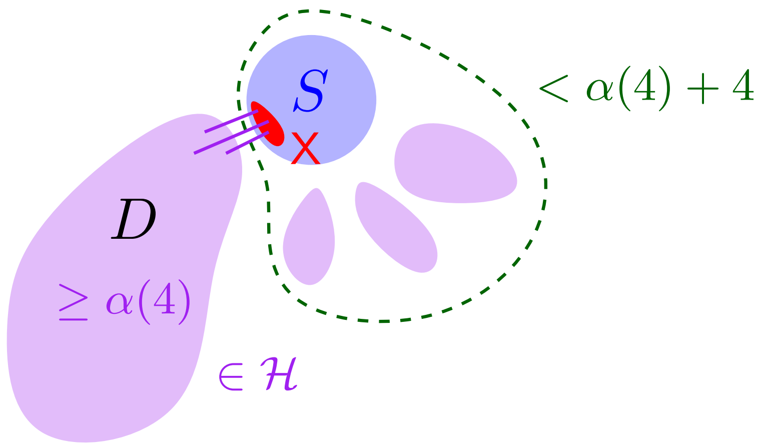

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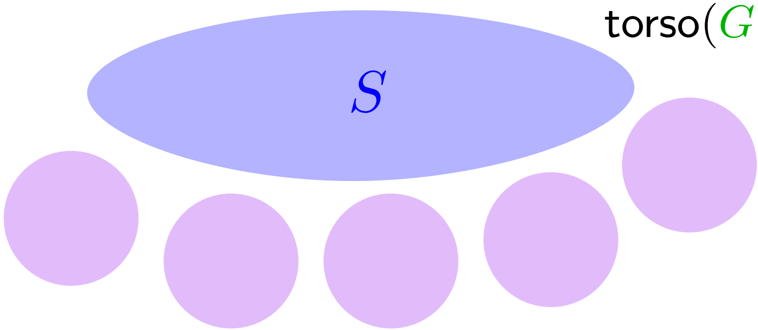
Guess the set $S \supseteq X$ in $G - D$ and check if the torso of S is planar and if the components of $G - S$ are in $\mathcal{H}$.

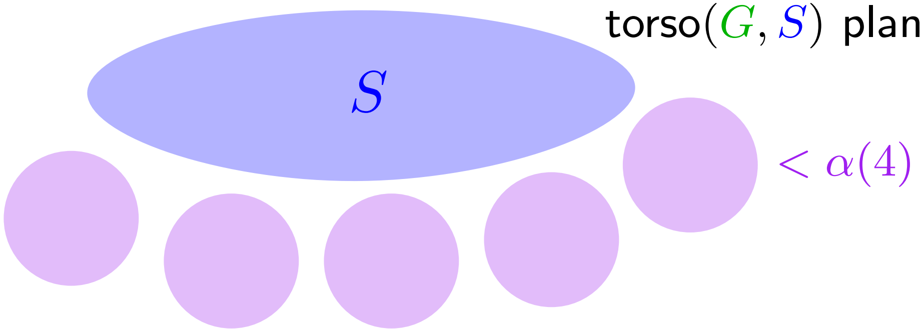
Sketch of the proof



torso(G, S) planar

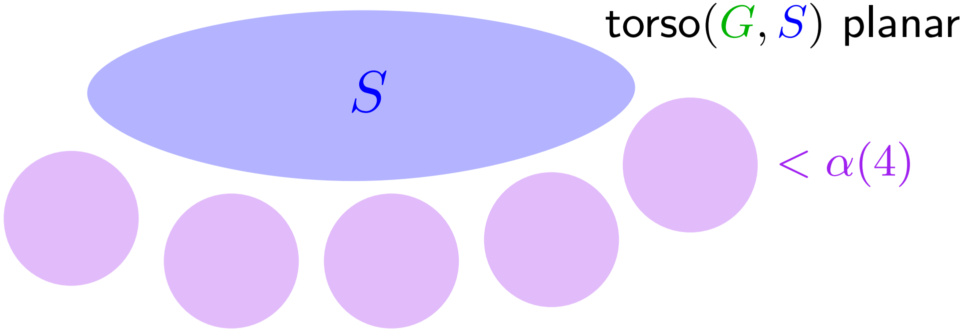
$< \alpha(4)$

Sketch of the proof



SMALL-LEAVES $\mathcal{H}$-PLANARITY

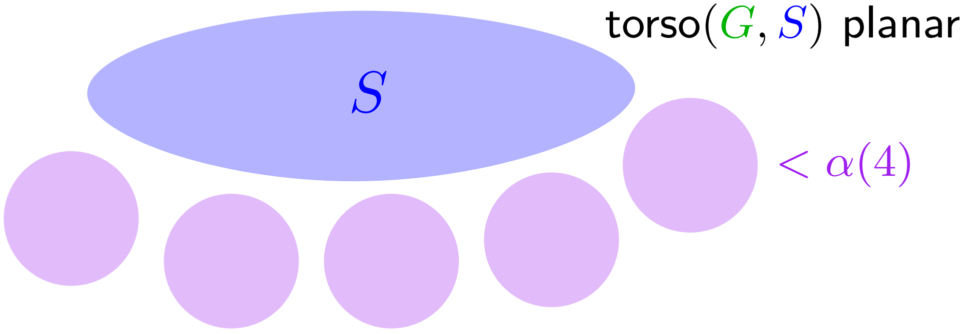
Sketch of the proof



Small-Leaves $\mathcal{H}$-Planarity

Restate the problem

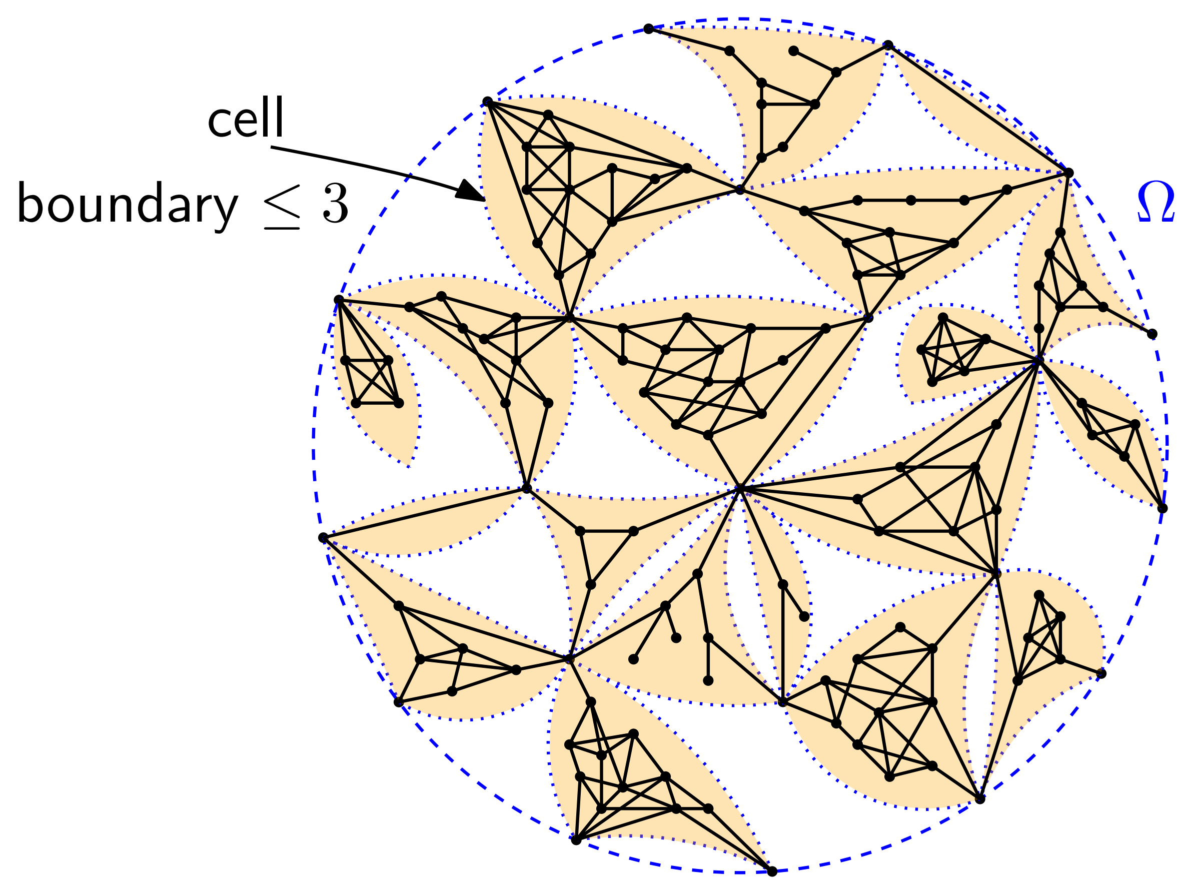
Sketch of the proof



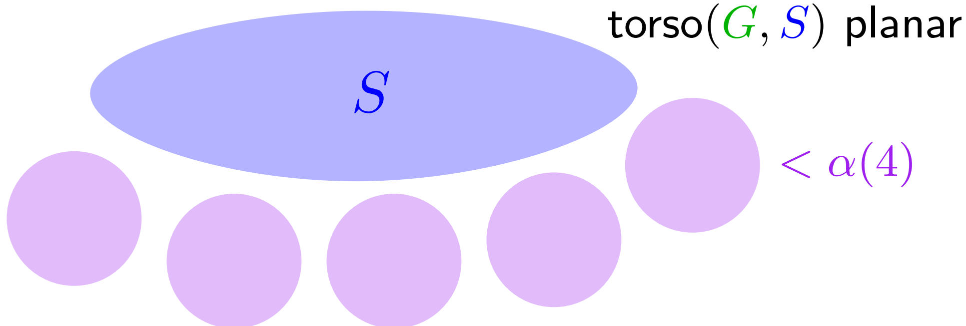
SMALL-LEAVES $\mathcal{H}$-PLANARITY

Restate the problem

Rendition of (G, Ω)



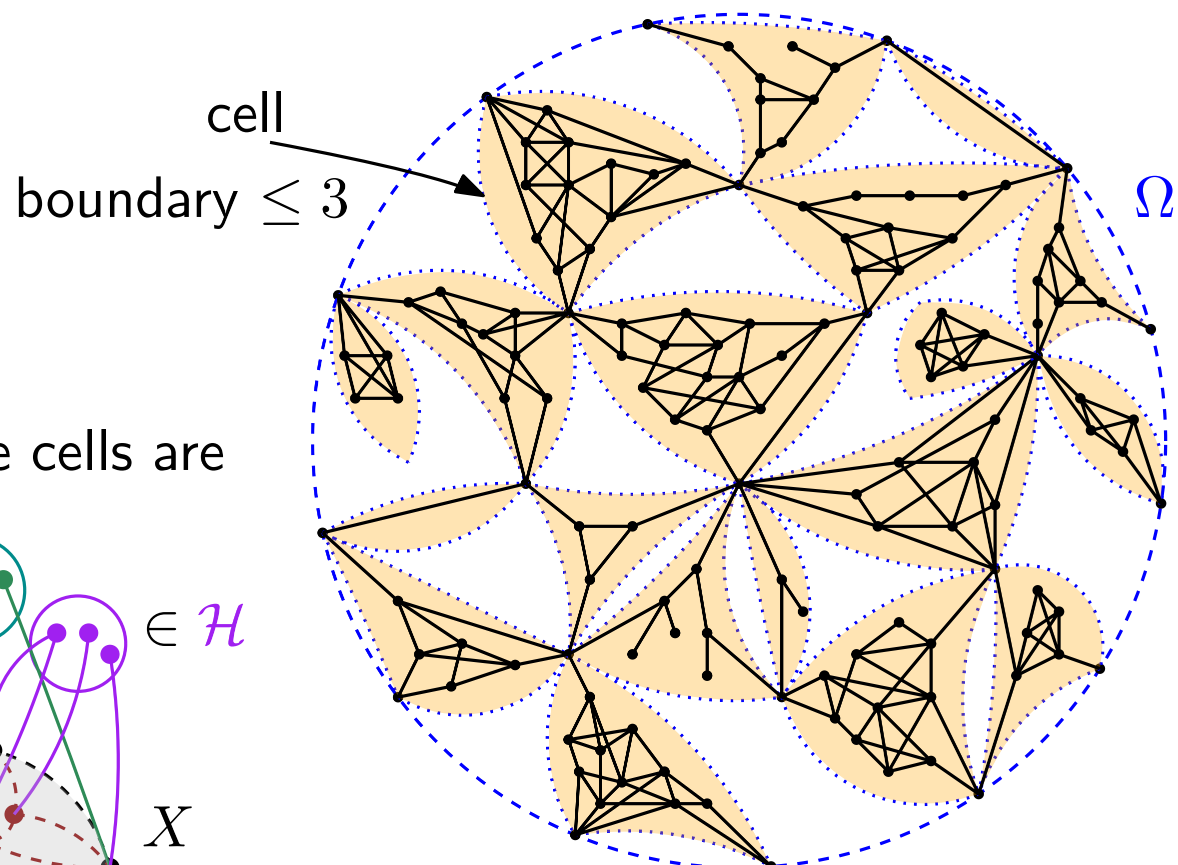
Sketch of the proof



Small-Leaves $\mathcal{H}$-Planarity

Restate the problem

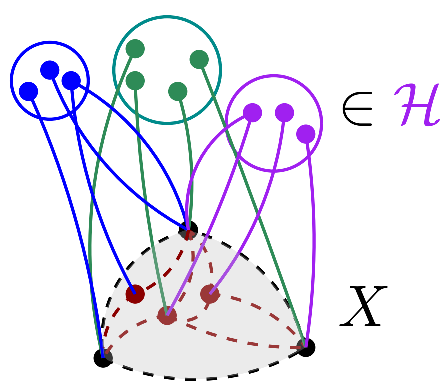
Rendition of (G, Ω)



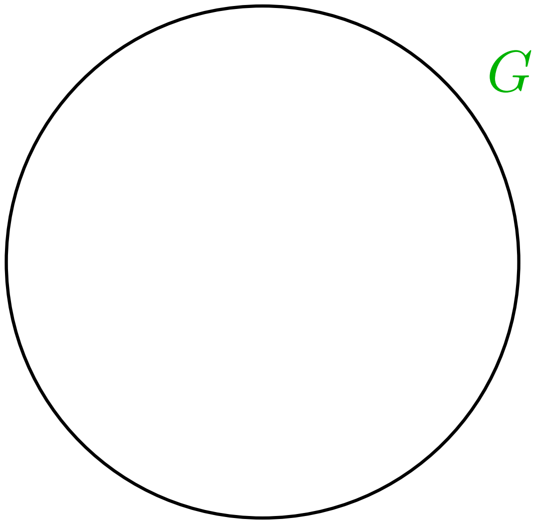
G is a yes-instance

$\Leftrightarrow$

G has a rendition whose cells are $\mathcal{H}$-**compatible**.

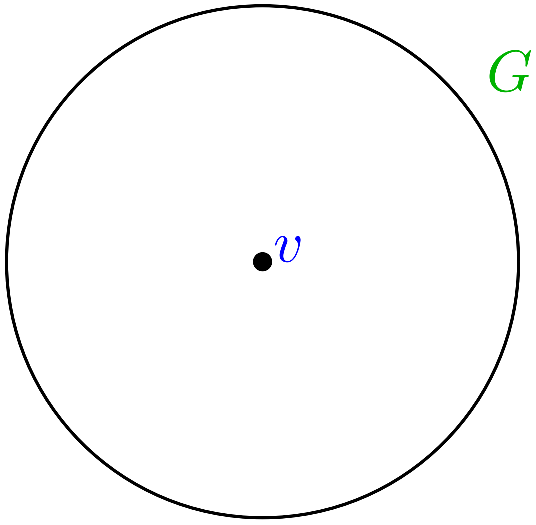


Idea:



Idea:

Pick a vertex v.

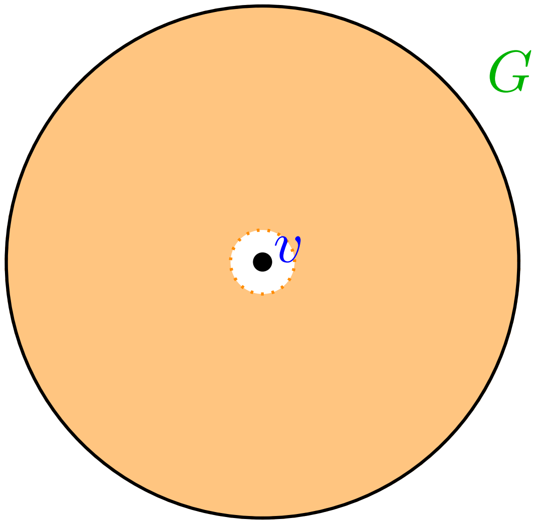


Idea:

Pick a vertex v.

Solve recursively on $G - v$.

Rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.



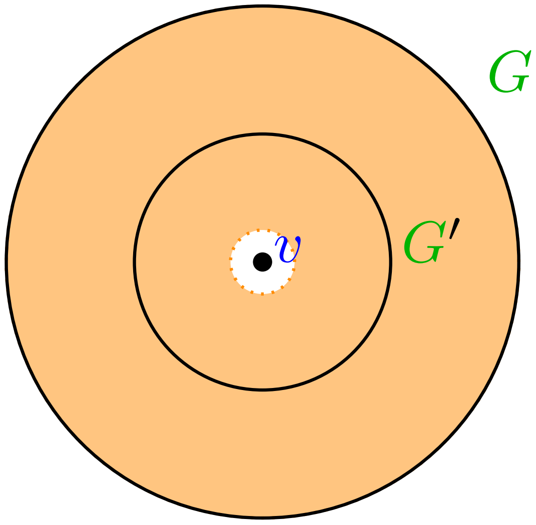
Idea:

Pick a vertex v.

Solve recursively on $G - v$.

Rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.



Idea:

Pick a vertex v.

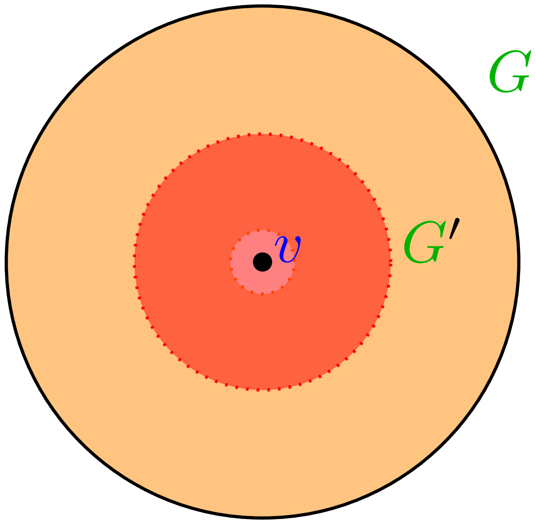
Solve recursively on $G - v$.

Rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Idea:

Pick a vertex v.

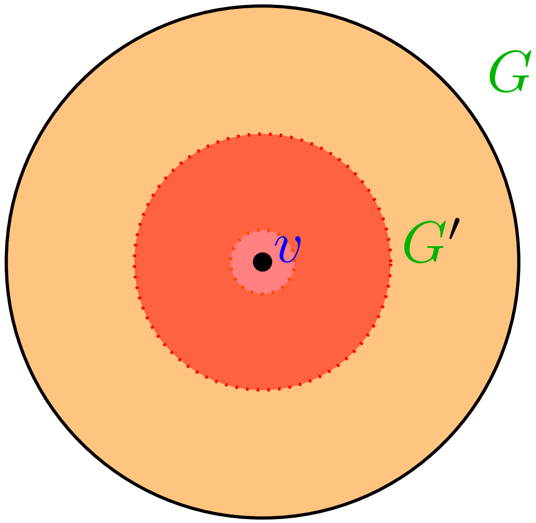
Solve recursively on $G - v$.

Rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



$\rightarrow$ want to combine ρ_1 and ρ_2 into a rendition of G whose cells are $\mathcal{H}$-compatible.

Idea:

Pick a vertex v.

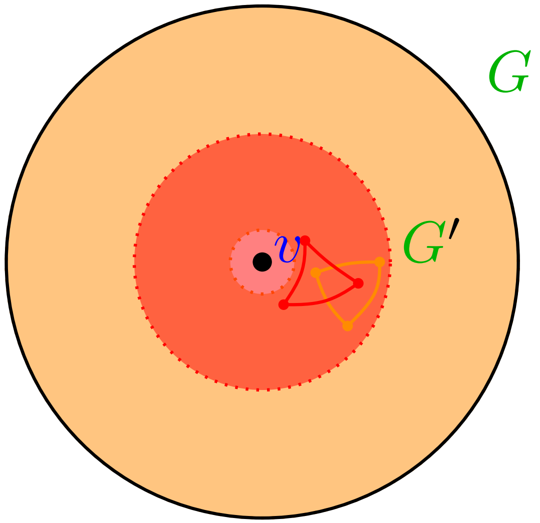
Solve recursively on $G - v$.

Rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

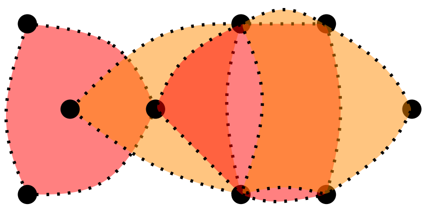
Solve on G'. [Courcelle, '90]

Rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.

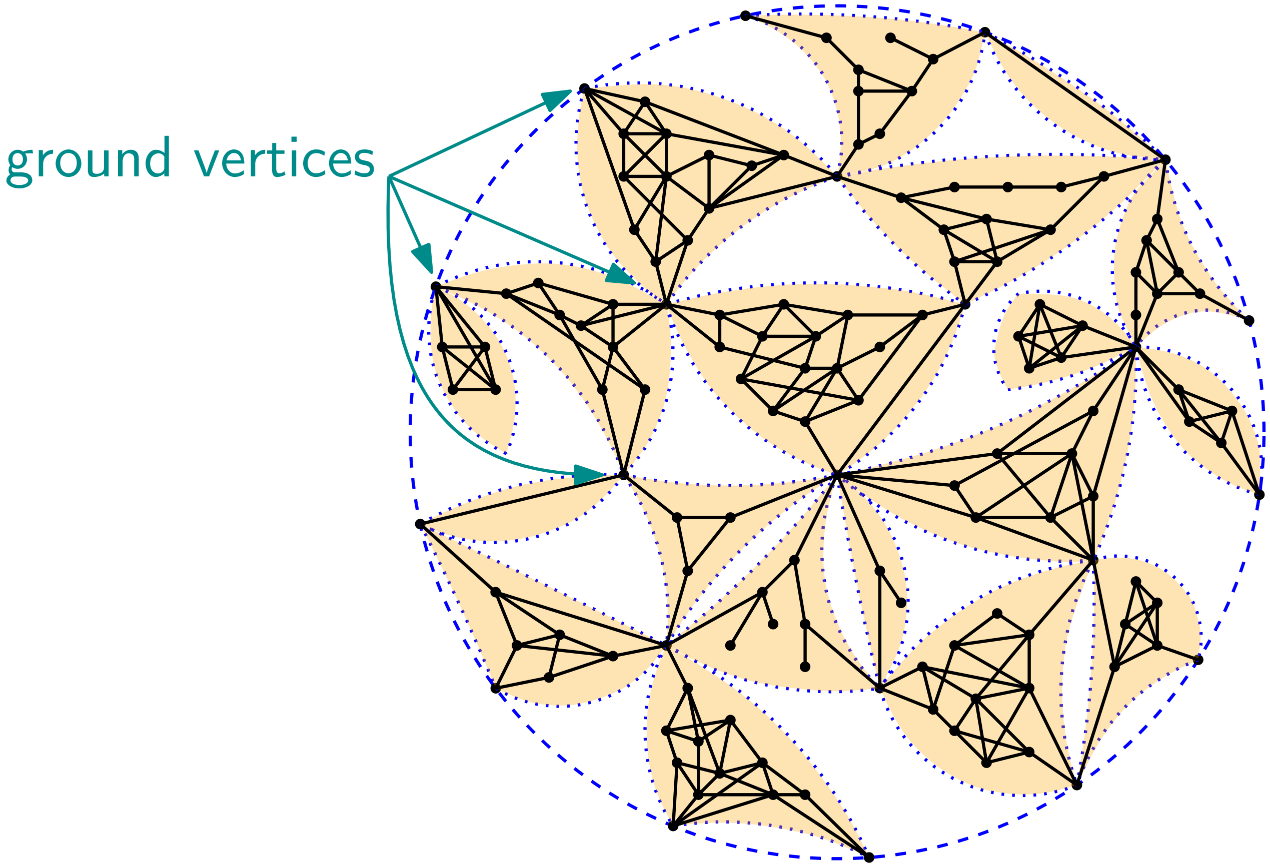


$\rightarrow$ want to combine ρ_1 and ρ_2 into a rendition of G whose cells are $\mathcal{H}$-compatible.

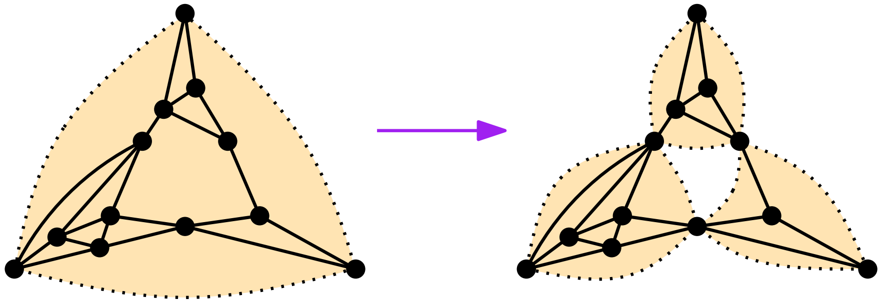
Problem: How to glue correctly?

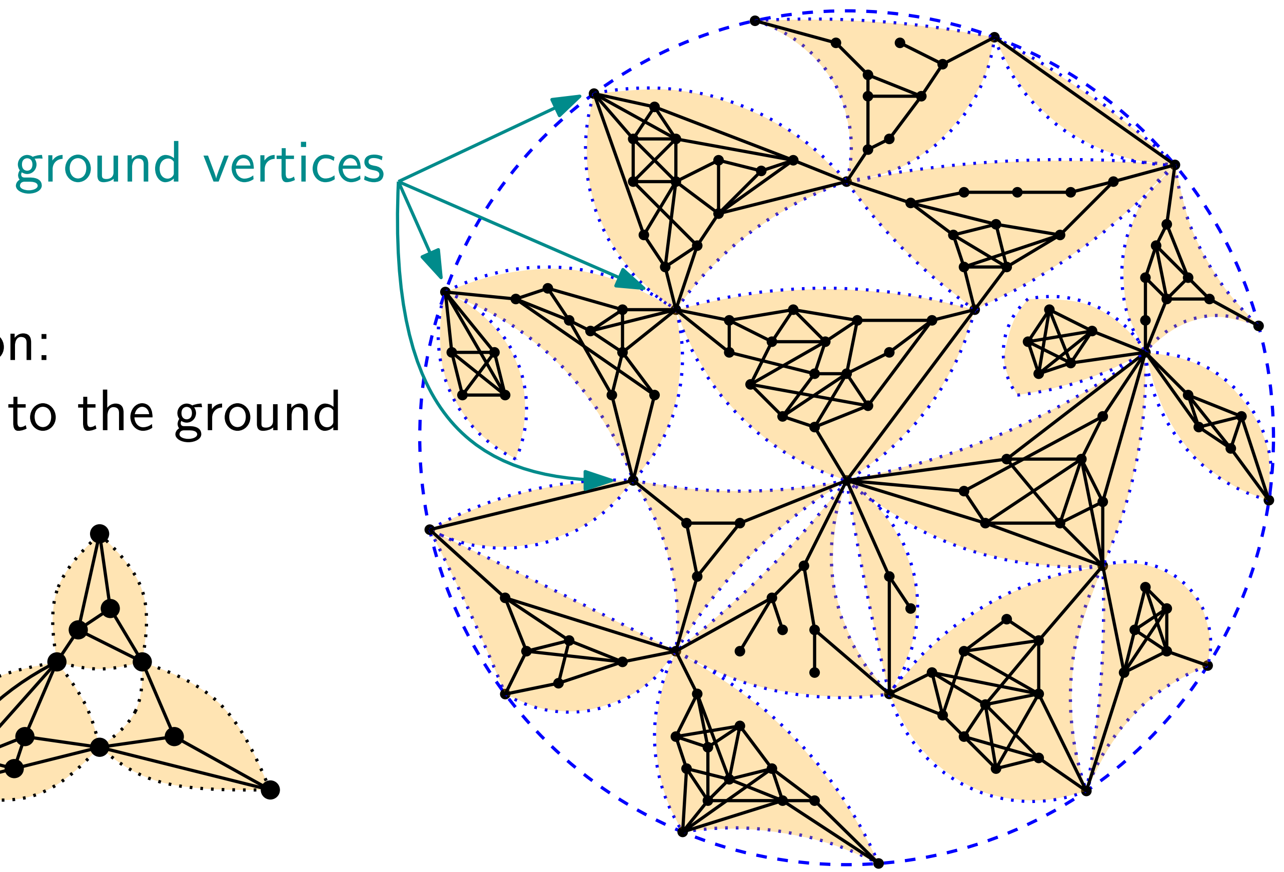


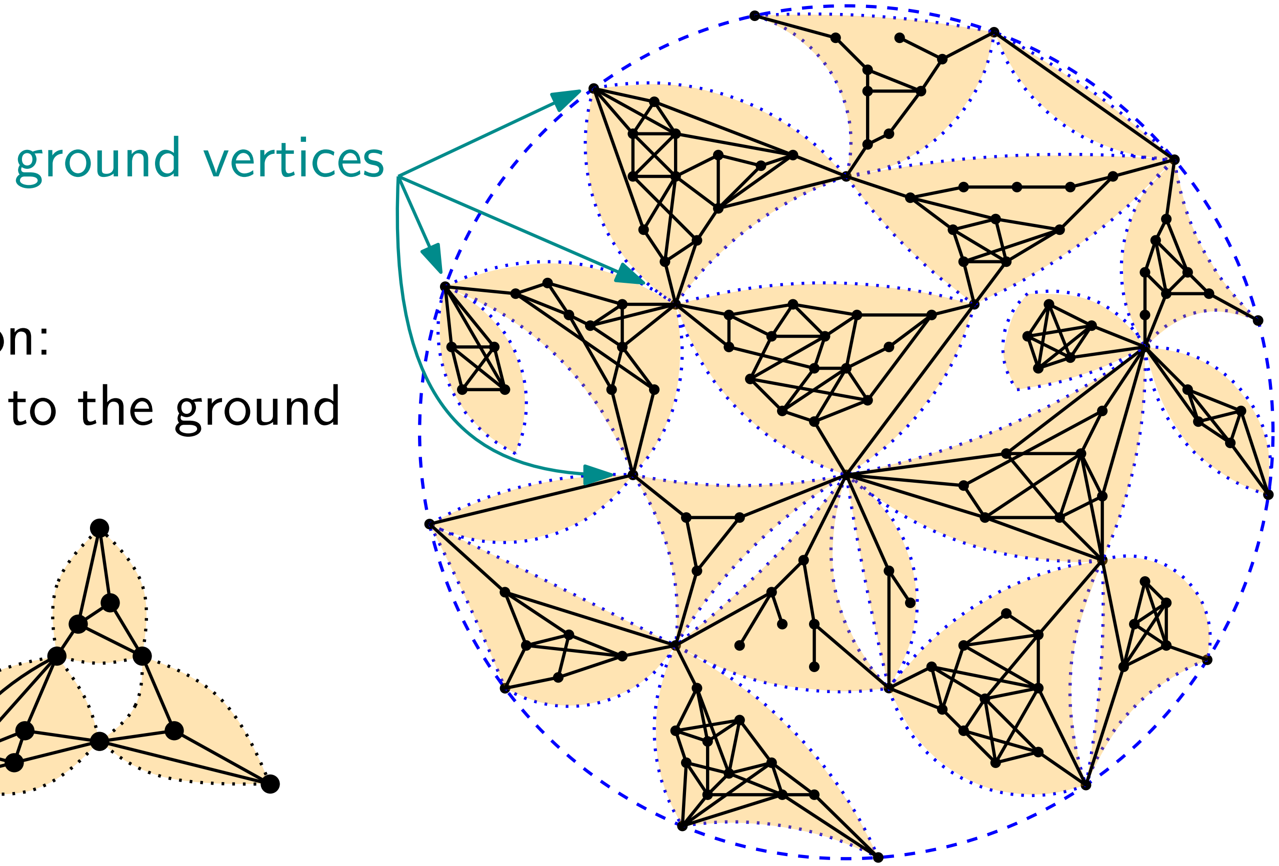
no "canonical rendition" of a graph


ground vertices

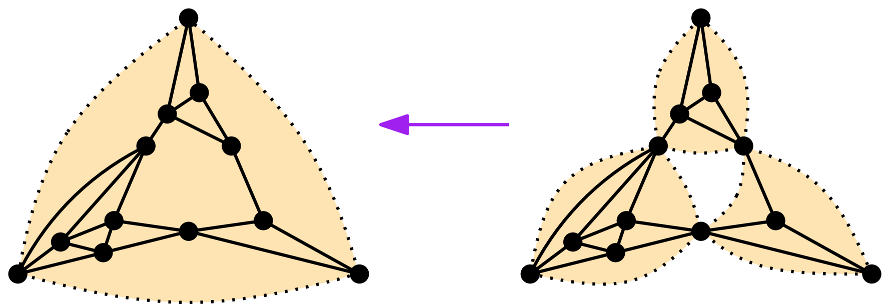
ground-maximal rendition:
cannot add more vertices to the ground



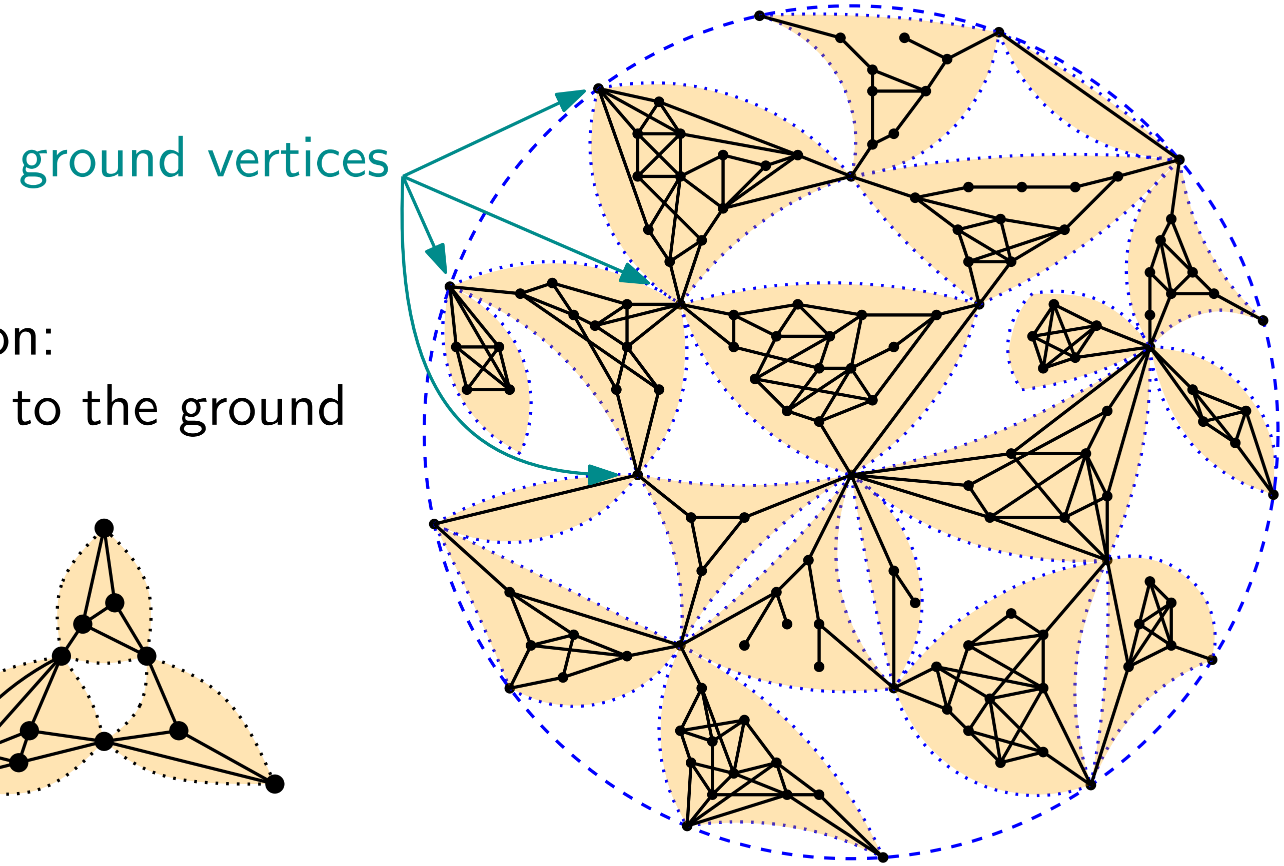




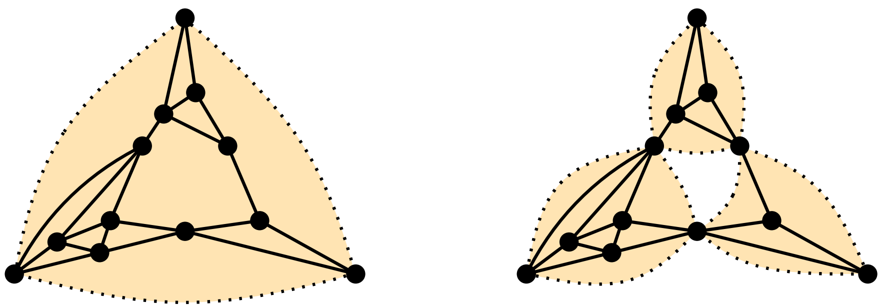
ground-maximal rendition:
cannot add more vertices to the ground



ground-minimal rendition:
cannot remove more vertices from the ground

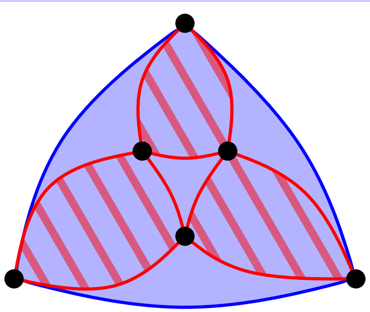


ground-maximal rendition:
cannot add more vertices to the ground



ground-minimal rendition:
cannot remove more vertices from the ground

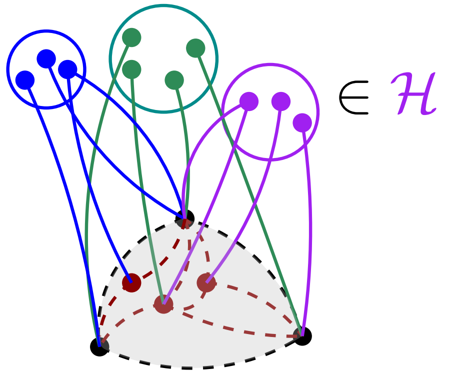
Every cell of ground-maximal rendition is contained in a cell of a ground-minimal rendition.

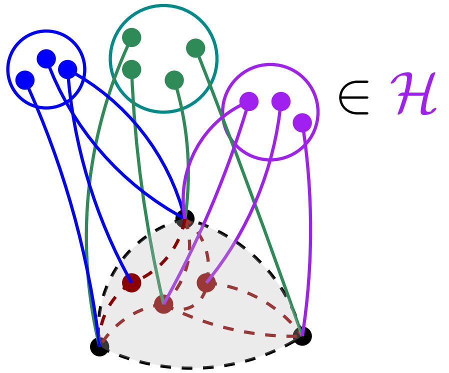


G is a yes-instance of Small-Leaves $\mathcal{H}$-Planarity

$\Leftrightarrow$

G has a rendition whose cells are $\mathcal{H}$-**compatible**.





G is a yes-instance of Small-Leaves $\mathcal{H}$-Planarity

$\Leftrightarrow$

G has a rendition whose cells are $\mathcal{H}$-**compatible**.

$\Leftrightarrow$

G has a ground-maximal rendition whose cells are $\mathcal{H}$-**compatible**.

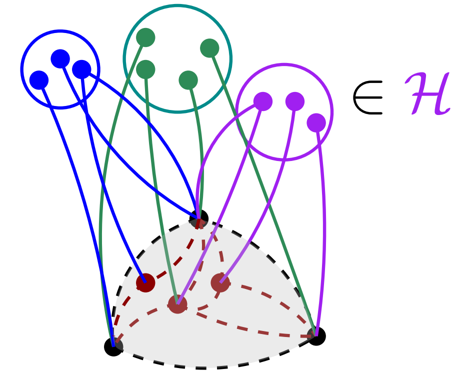
G is a yes-instance of Small-Leaves $\mathcal{H}$-Planarity

$\Leftrightarrow$

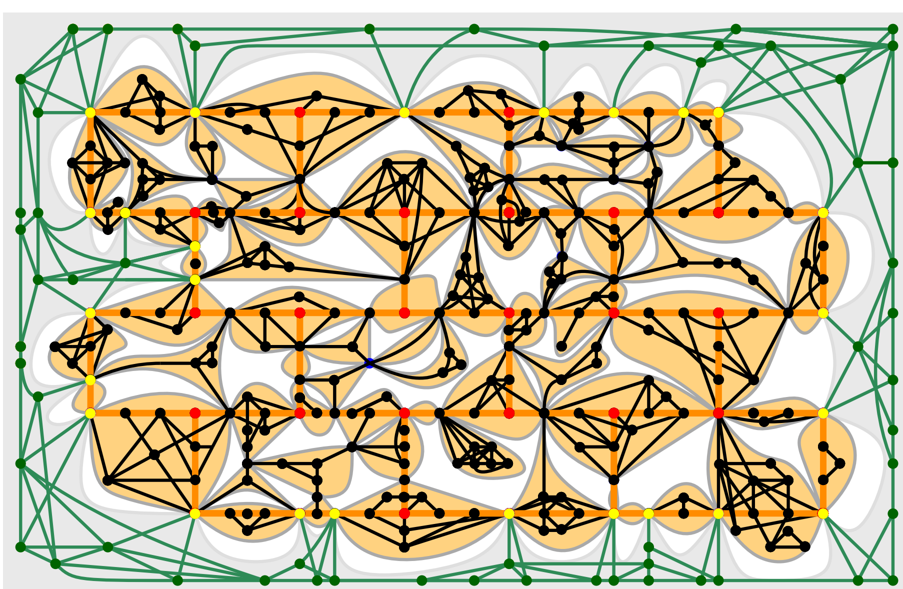
G has a rendition whose cells are $\mathcal{H}$-**compatible**.

$\Leftrightarrow$

G has a ground-maximal rendition whose cells are $\mathcal{H}$-**compatible**.



flat wall



[figure by Dimitrios M. Thilikos]

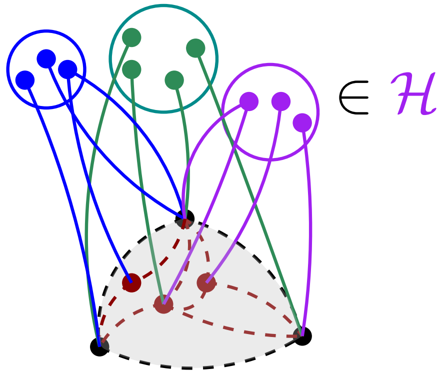
G is a yes-instance of SMALL-LEAVES $\mathcal{H}$-PLANARITY

$\Leftrightarrow$

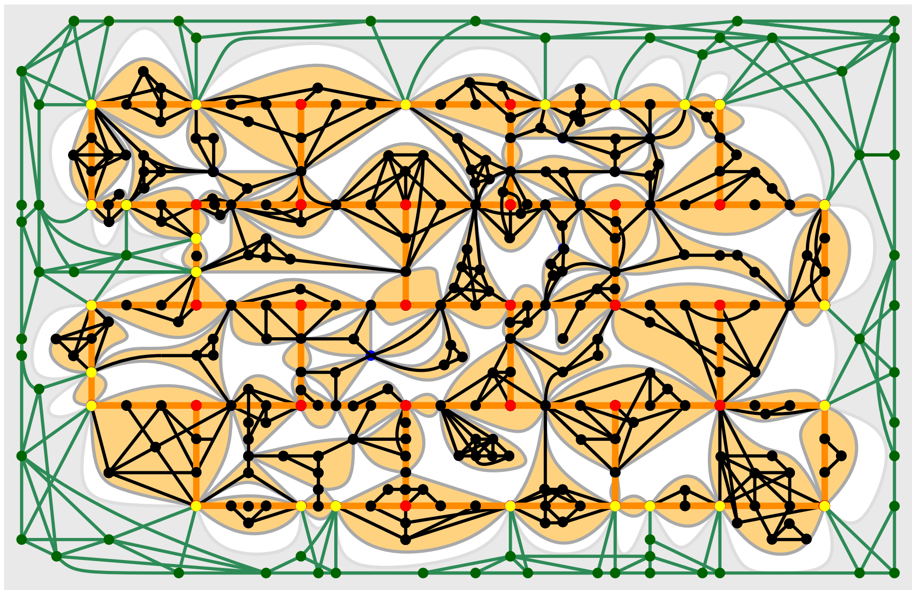
G has a rendition whose cells are $\mathcal{H}$-**compatible**.

$\Leftrightarrow$

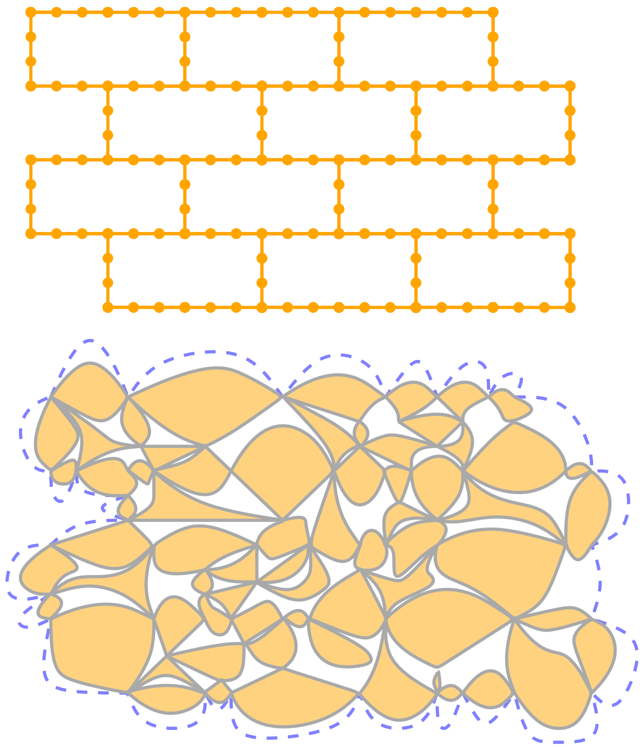
G has a ground-maximal rendition whose cells are $\mathcal{H}$-**compatible**.

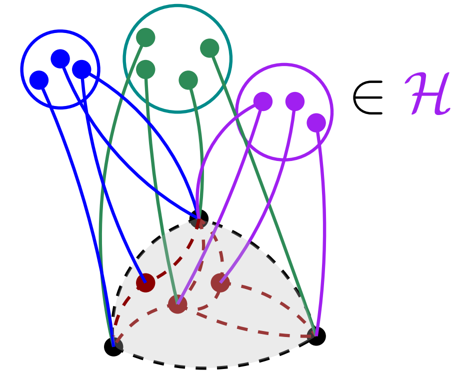


flat wall = wall + rendition



[figure by Dimitrios M. Thilikos]





G is a yes-instance of Small-Leaves $\mathcal{H}$-Planarity

$\Leftrightarrow$

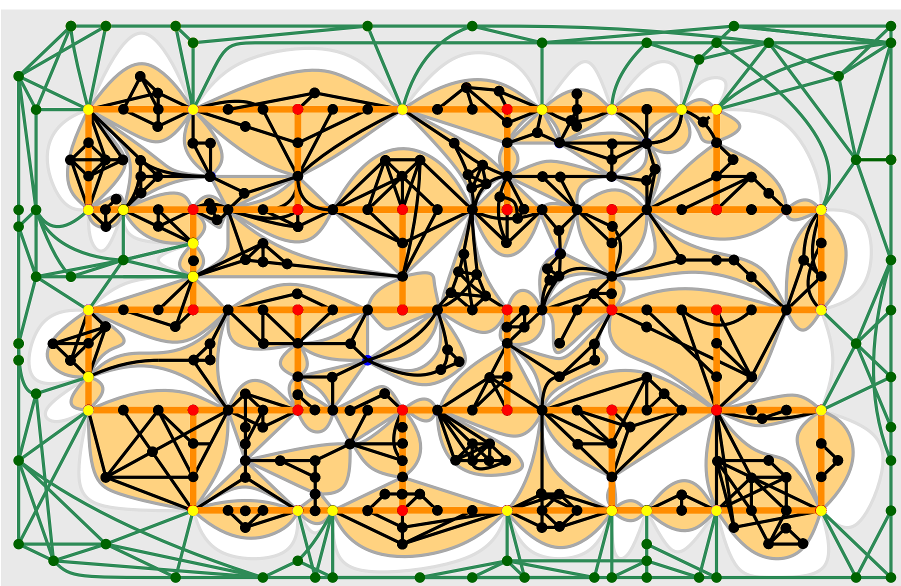
G has a rendition whose cells are $\mathcal{H}$-**compatible**.

$\Leftrightarrow$

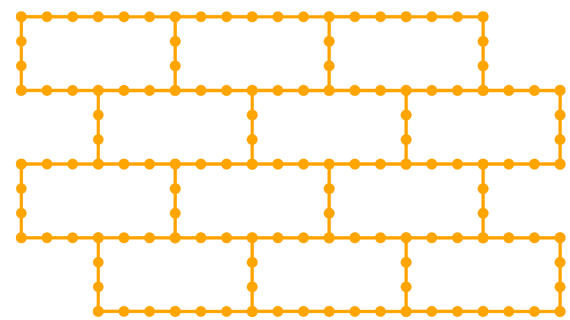
G has a ground-maximal rendition whose cells are $\mathcal{H}$-**compatible**.

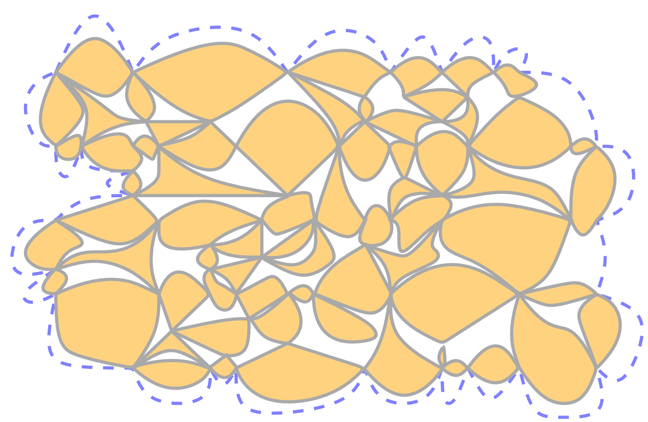
can choose ground-minimal

flat wall $=$ wall $+$ rendition



[figure by Dimitrios M. Thilikos]





Pick a vertex v

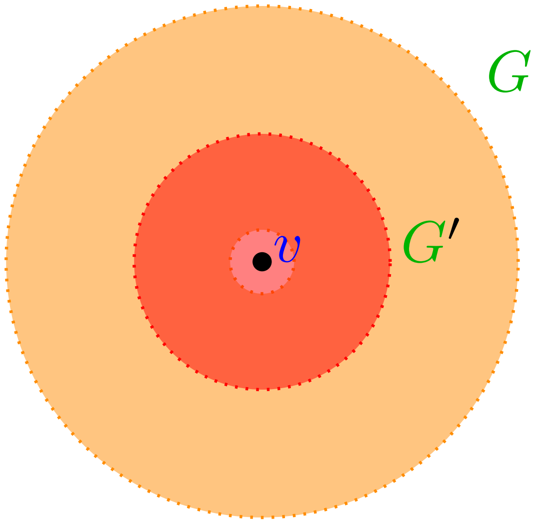
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v

Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

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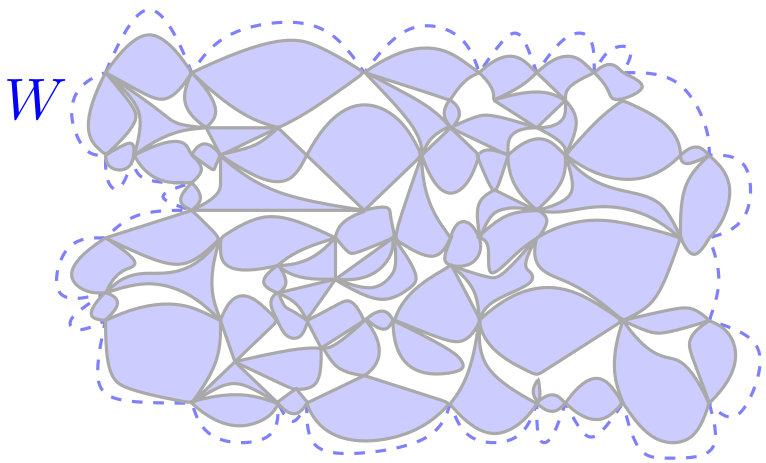
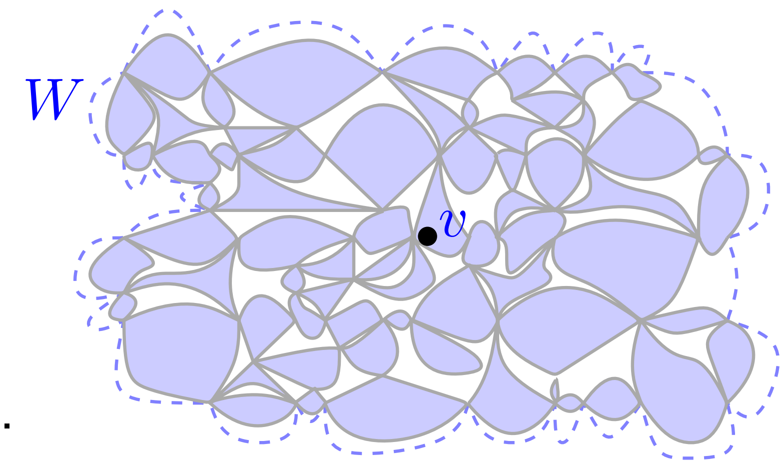
Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.



Solve recursively on $G - v$.

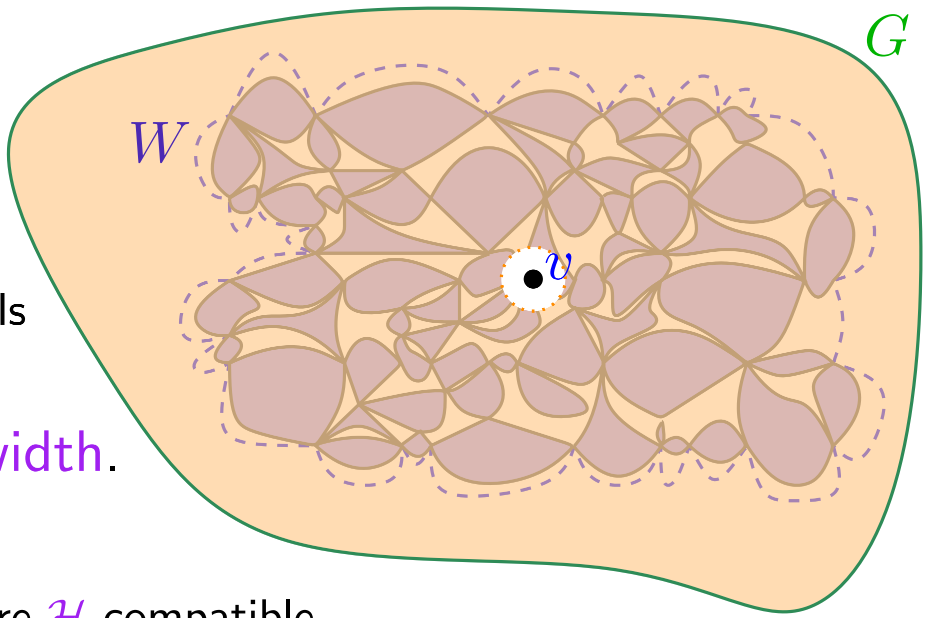
Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).



Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

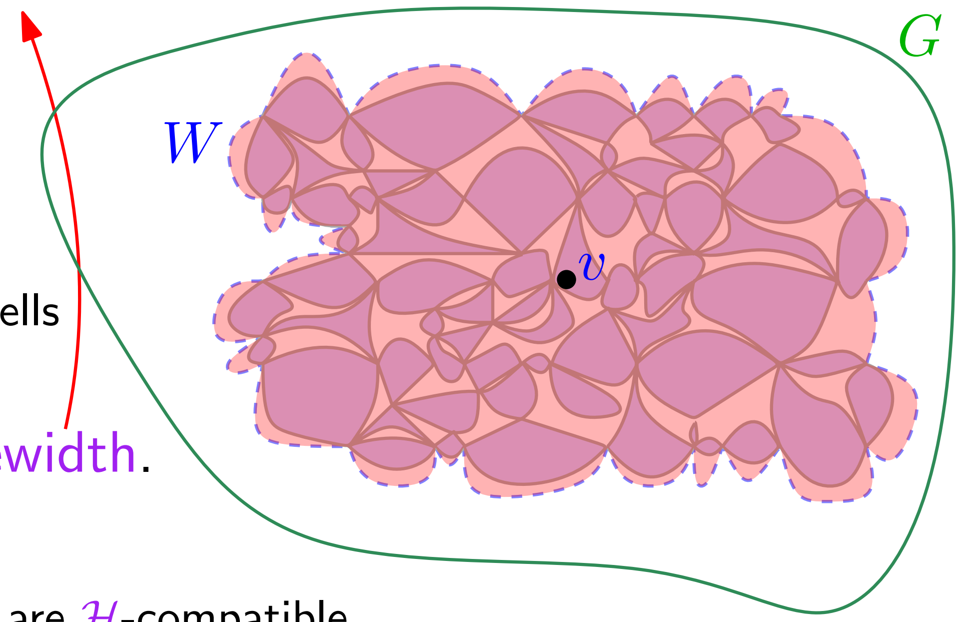
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

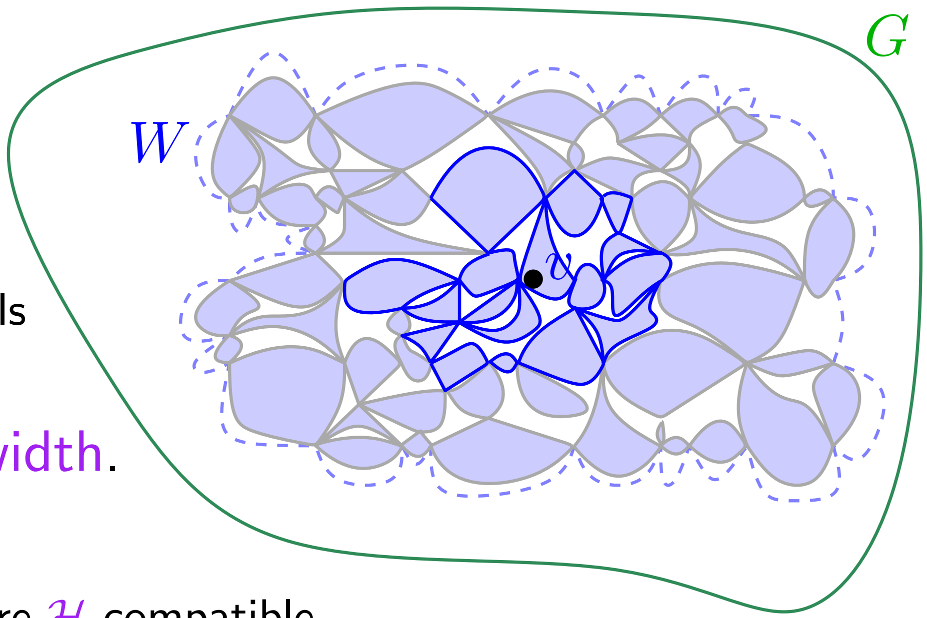
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Take a region R of cells of ρ' around v in W.

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

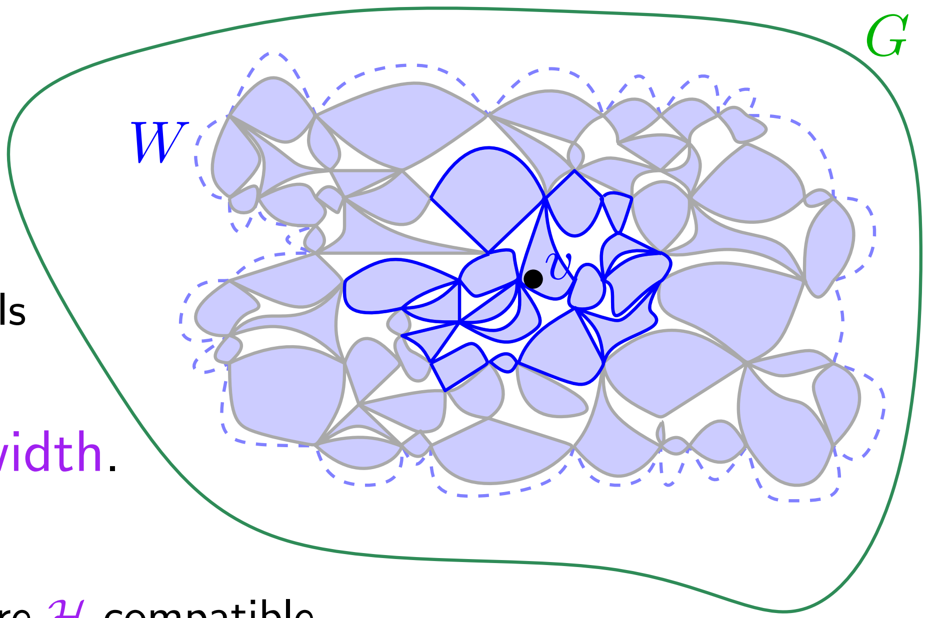
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

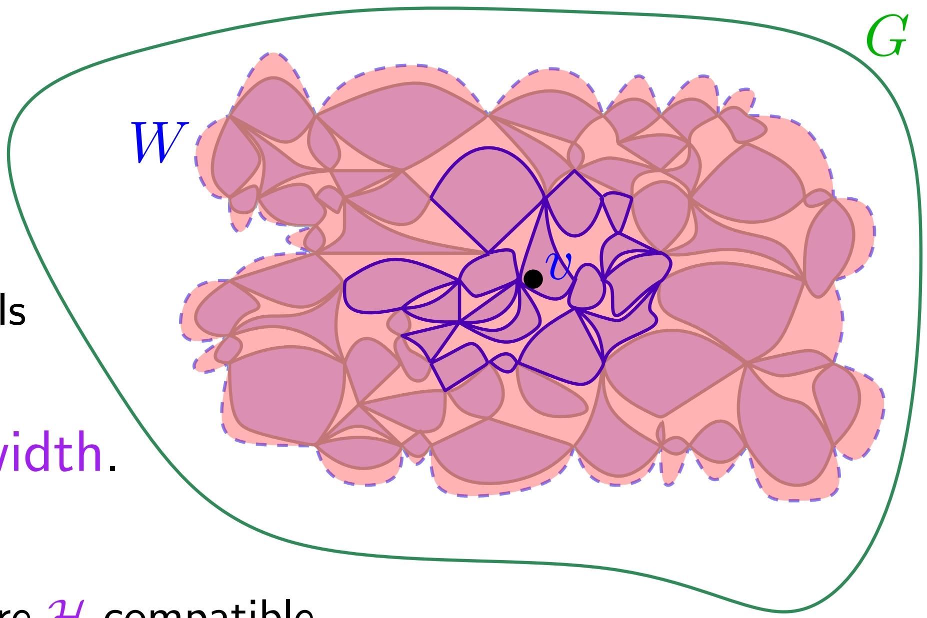
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

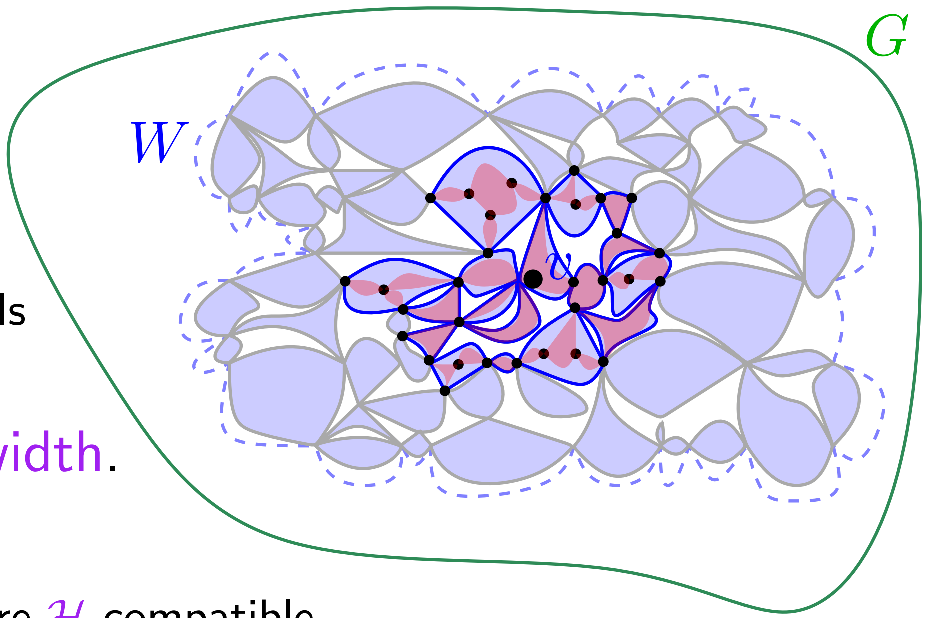
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

$\rightarrow$ replace the cells of ρ' in R with cells of ρ_2

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

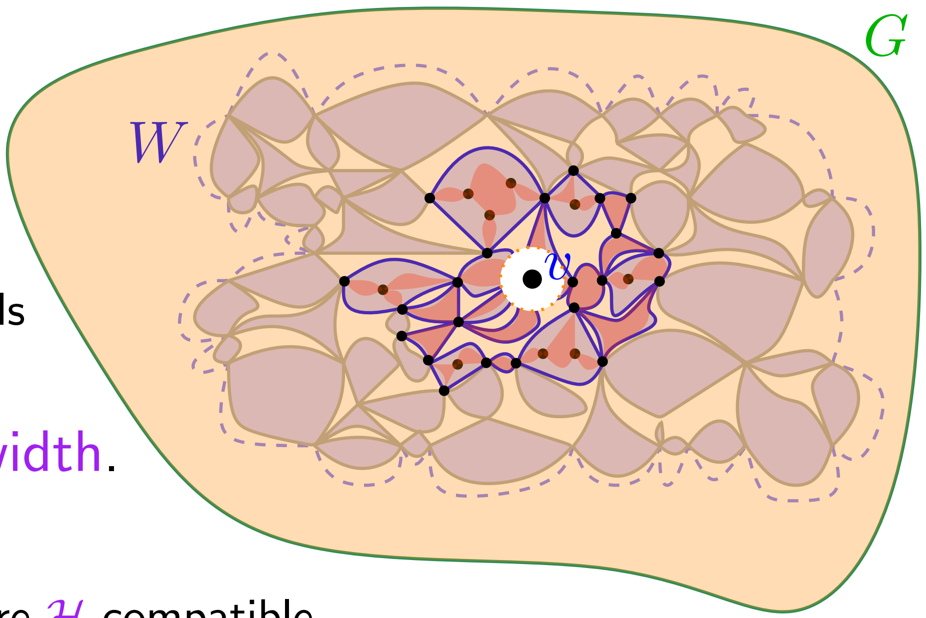
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



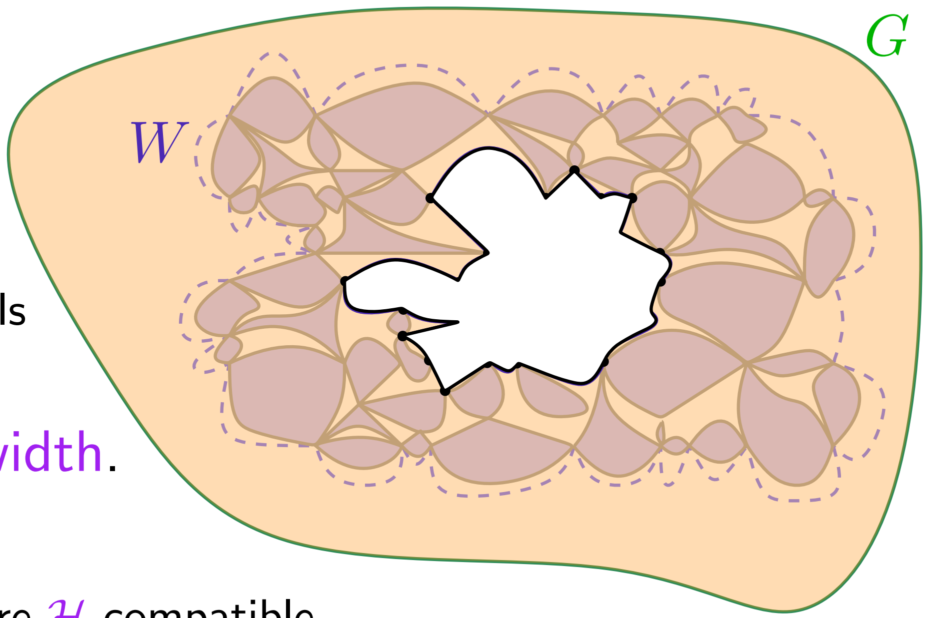
Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

$\rightarrow$ replace the cells of ρ' in R with cells of ρ_2

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.



Pick a vertex v in the center of W.

Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.

Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

$\rightarrow$ replace the cells of ρ' in R with cells of ρ_2

$\rightarrow$ replace the cells of ρ' outside of R with cells of ρ_1 (and use cells of ρ_1 outside of G')

Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

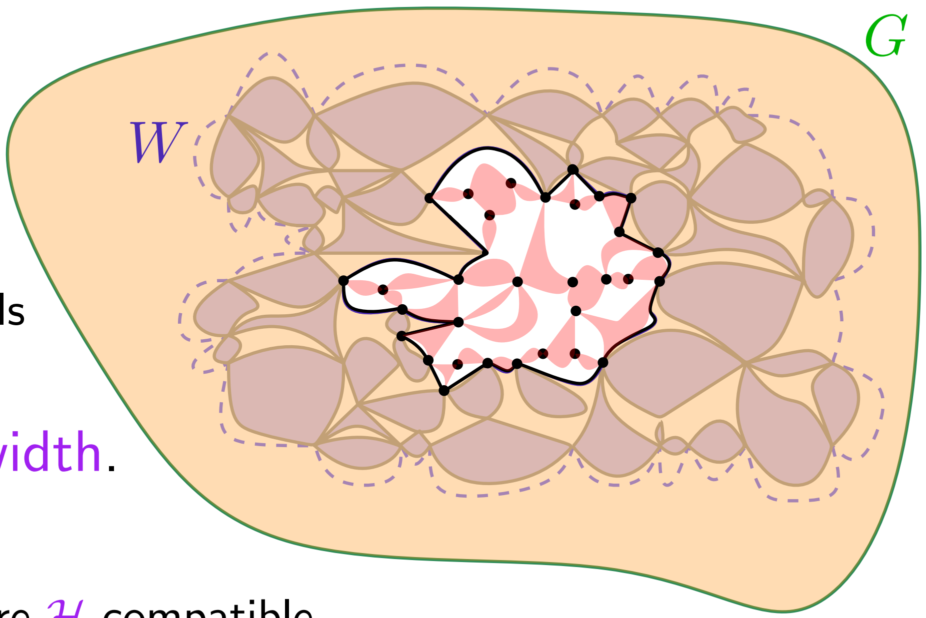
Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.

Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.



Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

$\rightarrow$ replace the cells of ρ' in R with cells of ρ_2

$\rightarrow$ replace the cells of ρ' outside of R with cells of ρ_1 (and use cells of ρ_1 outside of G')

$\Rightarrow$ rendition ρ of G whose cells are $\mathcal{H}$-compatible.

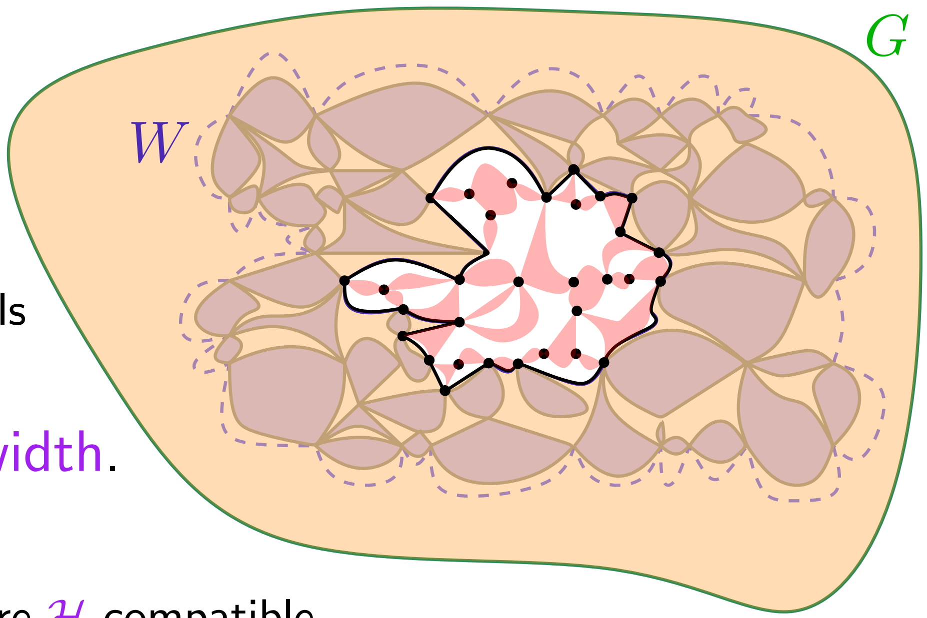
Find a flat wall W in G whose interior G' has bounded treewidth (or conclude).

Ground-minimal rendition ρ' in G'.

Pick a vertex v in the center of W.

Solve recursively on $G - v$.

Ground-maximal rendition ρ_1 of $G - v$ whose cells are $\mathcal{H}$-compatible.



Take a region G' around v of small treewidth.

Solve on G'. [Courcelle, '90]

Ground-maximal rendition ρ_2 of G' whose cells are $\mathcal{H}$-compatible.

Take a region R of cells of ρ' around v in W.

Each cell of ρ_1 and ρ_2 is contained in a cell of ρ'. $\rightarrow$ can glue ρ_1 and ρ_2 at the boundary of R

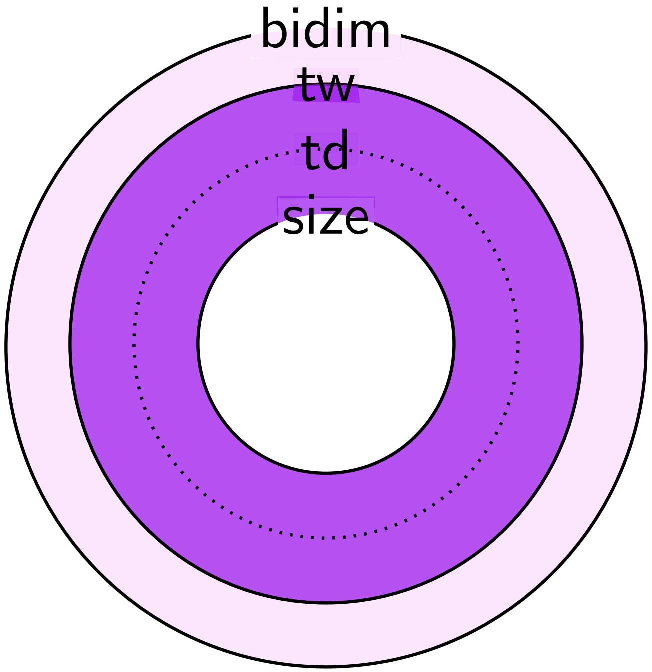
$\rightarrow$ replace the cells of ρ' in R with cells of ρ_2

$\rightarrow$ replace the cells of ρ' outside of R with cells of ρ_1 (and use cells of ρ_1 outside of G')

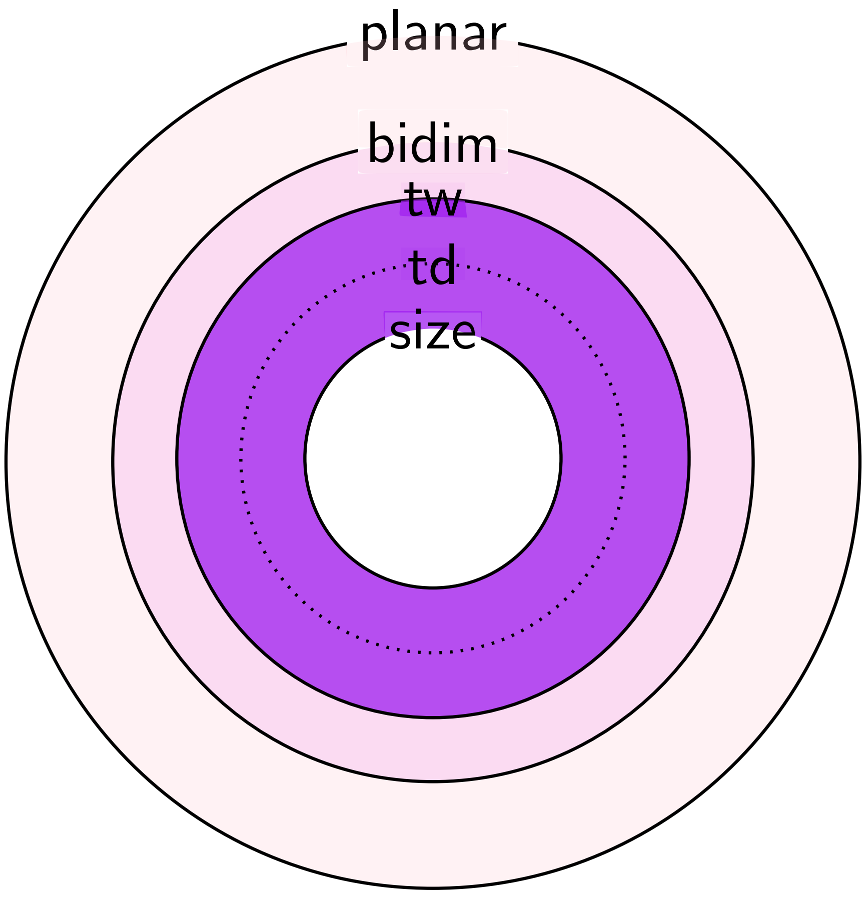
$\Rightarrow$ rendition ρ of G whose cells are $\mathcal{H}$-compatible.

$\Rightarrow$ G is a yes-instance.

Going even further



Going even further



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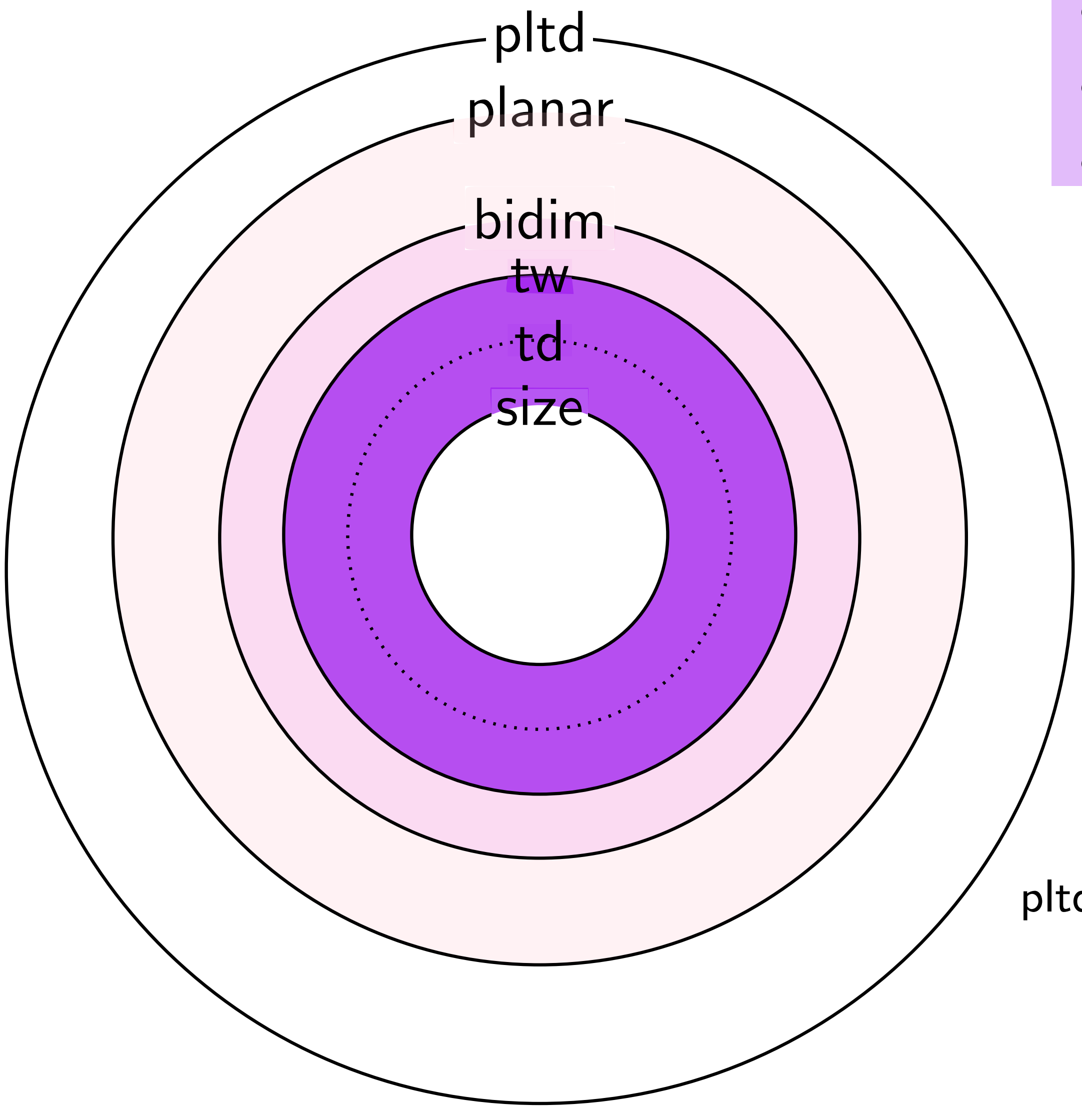

Graph class $\mathcal{H}$

- hereditary,
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[Fomin, Golovach, Morelle, Thilikos]

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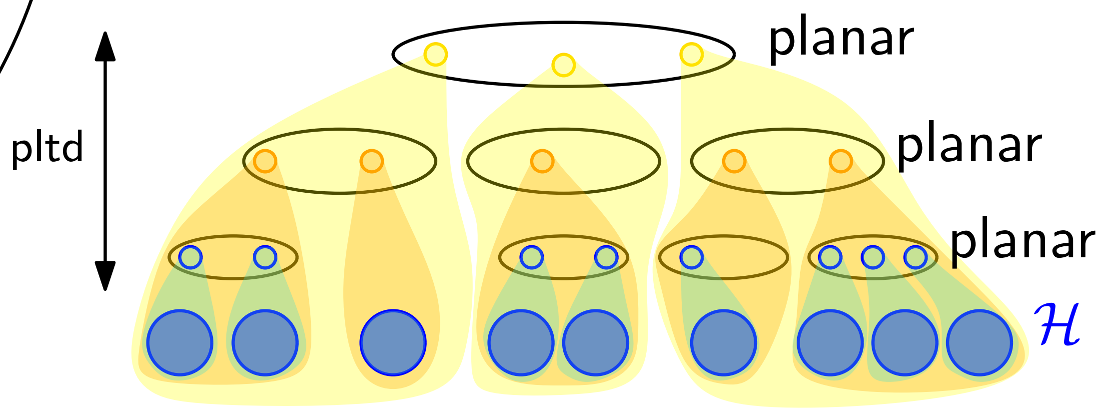
Planar treedepth pltd



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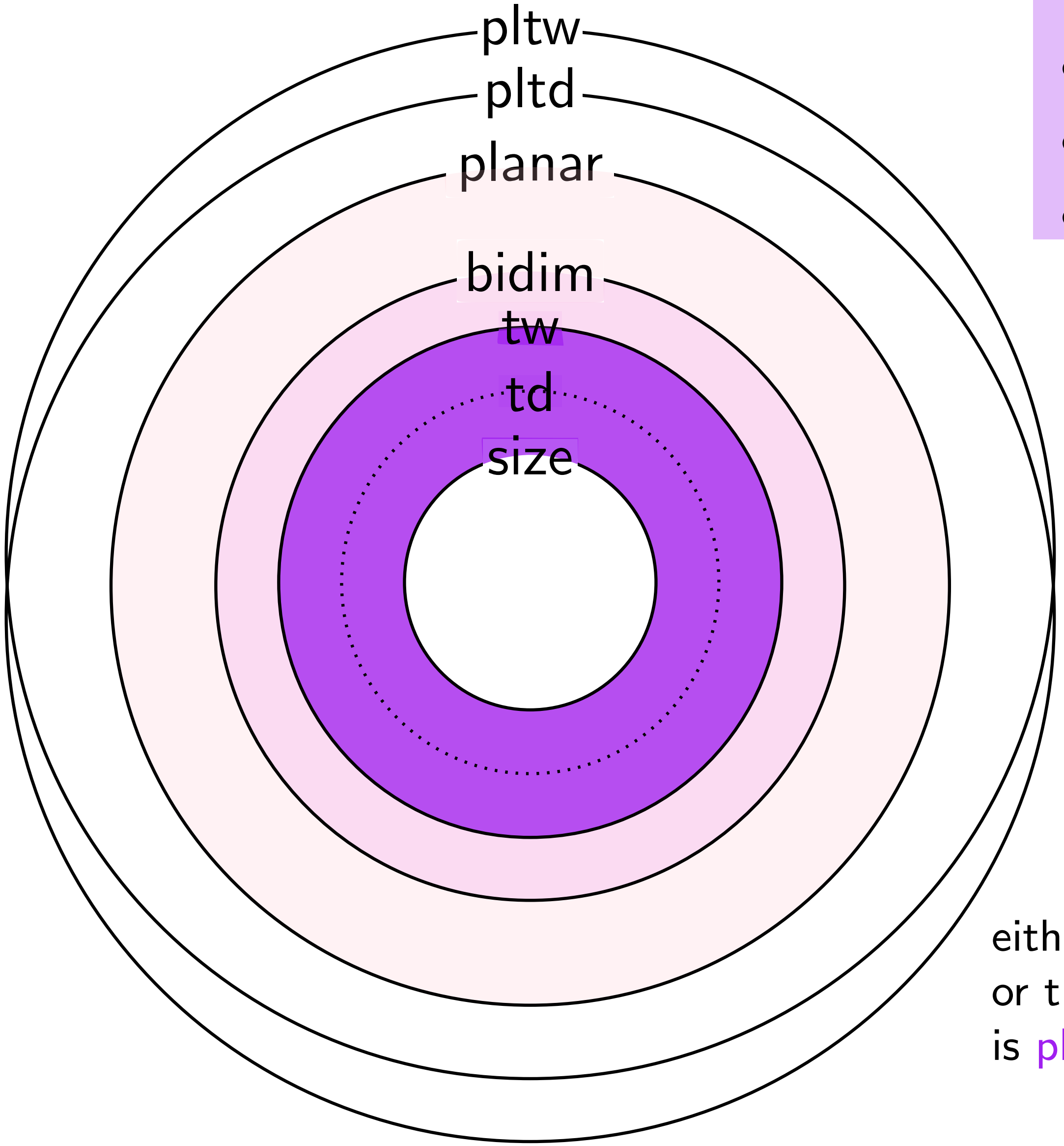

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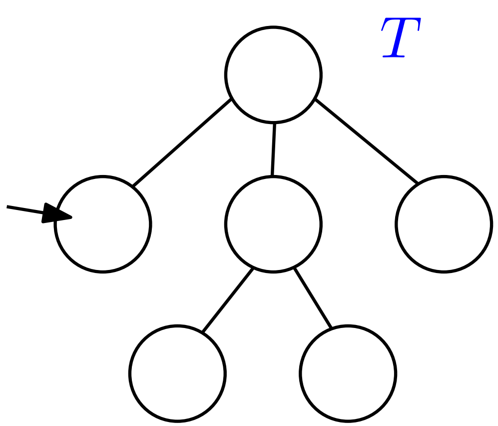
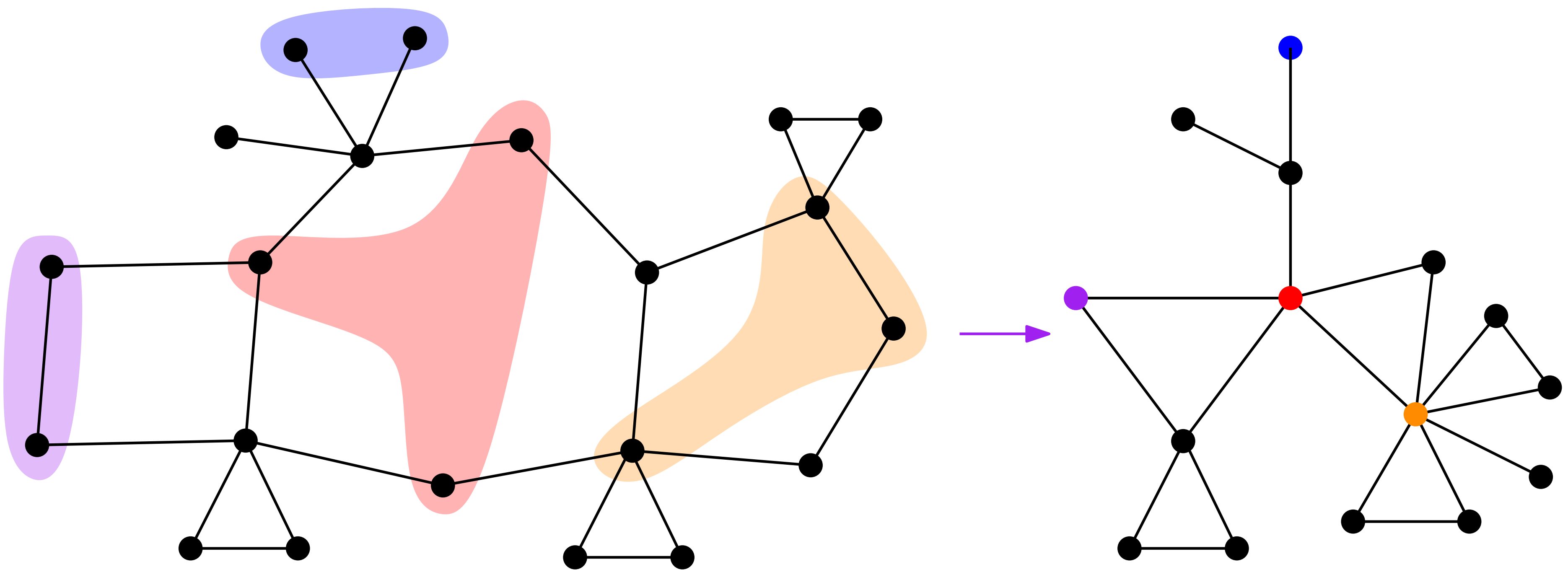
Planar treewidth pltw

Tree decomposition $(T, \{B_t\}_t)$ of G



4. A new modification: vertex identification

Vertex identification



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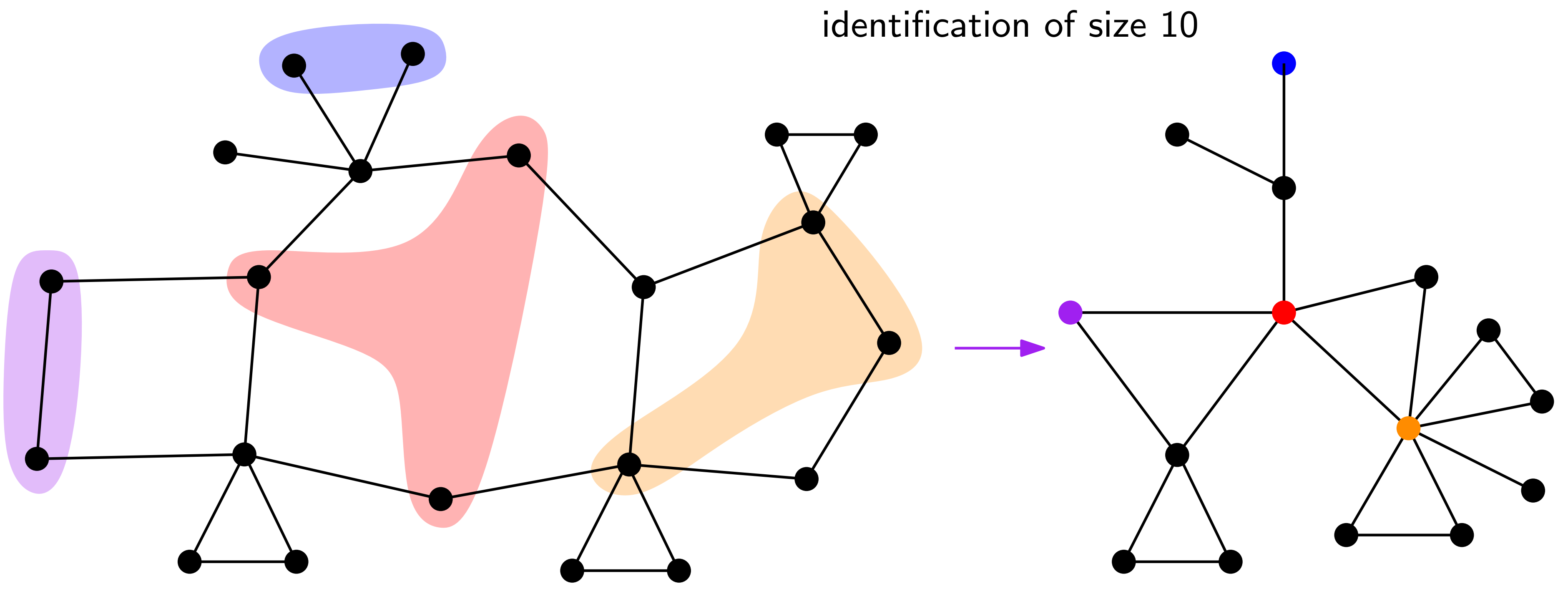

Size of the identification = number of vertices involved in the identification

Why identifications?

Structure theorems meet graph modification problems

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[Robertson, Seymour, '03]+[Thilikos, Wiederrecht, '23]

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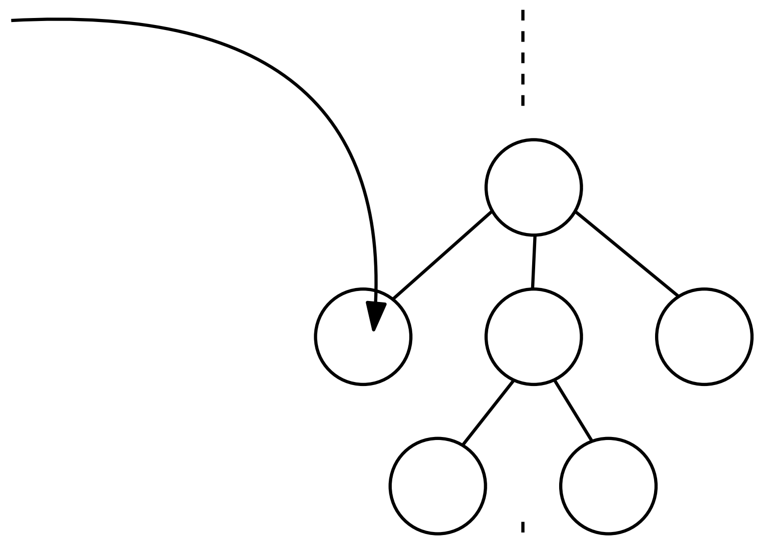
the torso of each bag

$h = |V(H)|$



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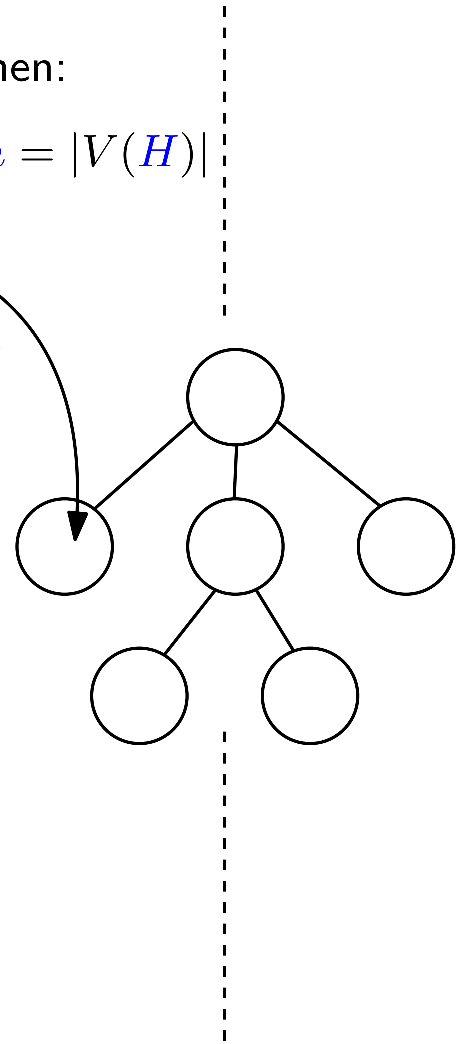

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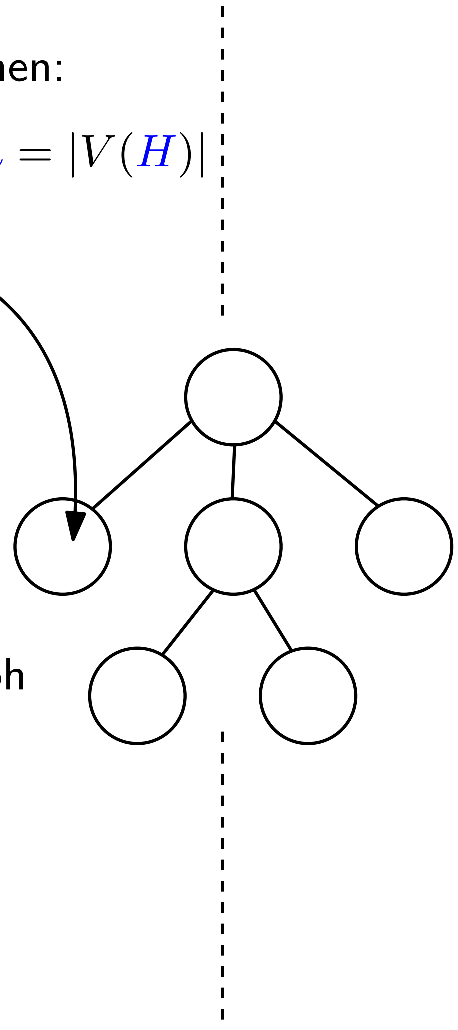

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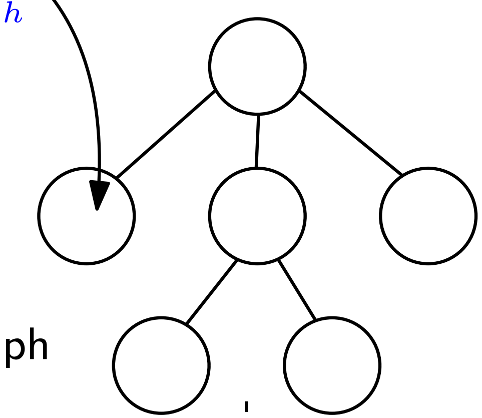

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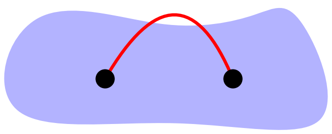

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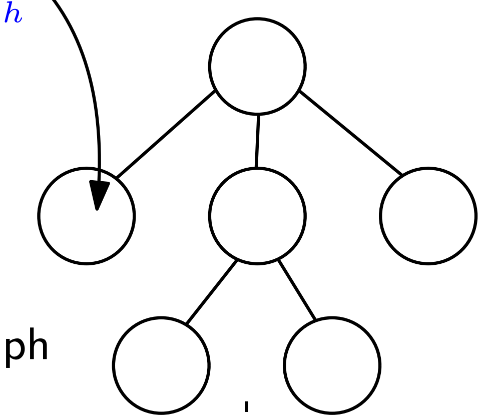

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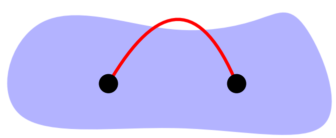

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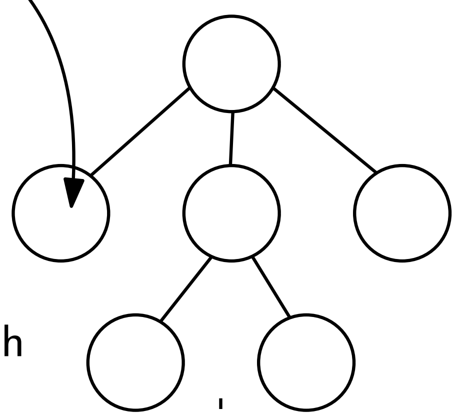

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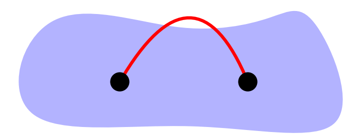

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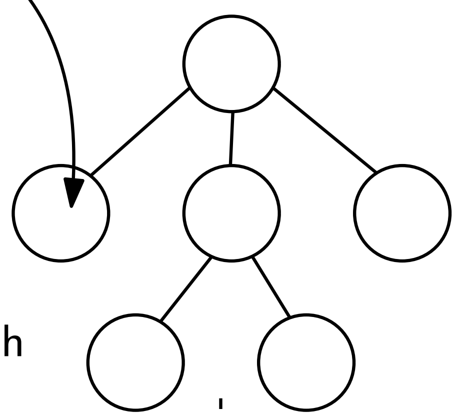

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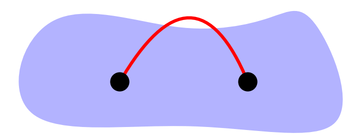

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Results on identifications

[Morelle, Sau, Thilikos]

VERTEX IDENTIFICATION TO FORESTS is solvable in time $\mathcal{O}(1.2738^k + k\sqrt{\log k} \cdot n)$.

[Morelle, Sau, Thilikos]

If $\mathcal{H}$ is minor-closed, then $\mathcal{L}$-REPLACEMENT TO $\mathcal{H}$ is solvable in time $2^{\text{poly}_{\mathcal{H}}(k)} \cdot n^2$ for $\mathcal{L}$ hereditary.

$\rightarrow$ includes VERTEX IDENTIFICATION TO $\mathcal{H}$

Further research

Direction 1: Efficiency

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Can we improve the running time of the different algorithms?

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[Korhonen, Pilipczuk, Stamoulis, '24]

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$$2^{k^{\mathcal{O}_{\mathcal{H}}(1)}} \cdot n^2 \longrightarrow n^{1+o(1)}?$$ [Korhonen, Pilipczuk, Stamoulis, '24]

$$\longrightarrow 2^{\mathcal{O}_{\mathcal{H}}(k^c)}?$$

Direction 2: Generalization

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Graph class $\mathcal{H}$ hereditary, closed under disjoint union, and CMSO-definable.
Parameter p minor-monotone.

for each minor H of G, $\mathsf{p}(H) \leq \mathsf{p}(G)$

Conjecture: If Vertex Deletion to $\mathcal{H}$ is FPT, then checking $\mathcal{H}$-$\mathsf{p}(G) \leq k$ is also FPT.

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Proved for $\mathsf{p} \in \{\mathsf{td}, \mathsf{tw}\}$ → likely to hold for any p with $\mathsf{tw} \leq \mathsf{p} \leq \mathsf{size}$.
[Agrawal, Kanesh, Lokshtanov, Panolan, Ramanujan, Saurabh, Zehavi, '22]
and $\mathsf{p} \in \{\mathsf{pltd}, \mathsf{pltw}\}$ → extension for any p?
[Fomin, Golovach, Morelle, Thilikos]

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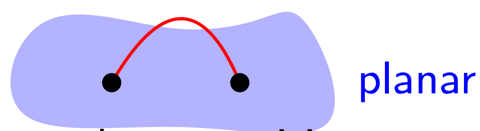

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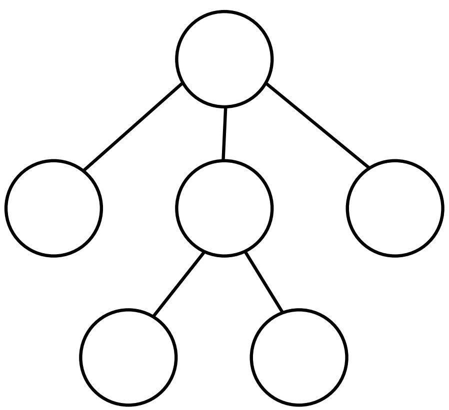

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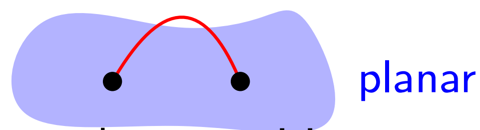

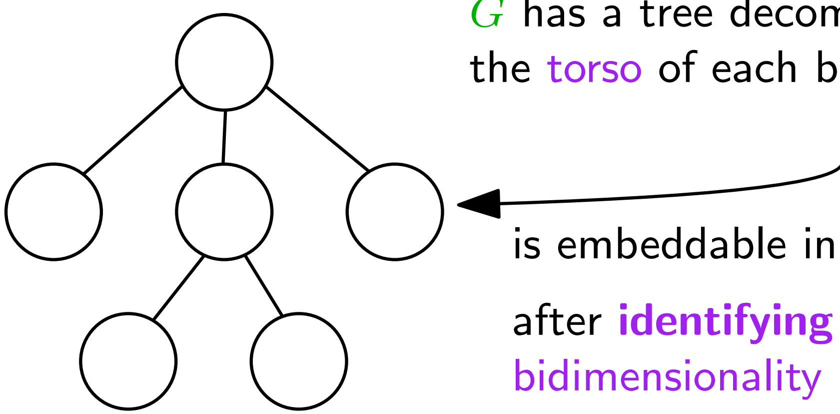

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Thank you!

- **Faster parameterized algorithms for modification problems to minor-closed classes**, with Ignasi Sau, Giannos Stamoulis, and Dimitrios M. Thilikos. ICALP 2023, TheoretiCS 2024.
- Dynamic programming on bipartite tree decompositions, with Lars Jaffke, Ignasi Sau, and Dimitrios M. Thilikos. IPEC 2023, submitted to a journal.
- PACE Solver Description: Touiouidth, with Gaétan Berthe, Yoann Coudert–Osmont, Alexander Dobler, Amadeus Reinald, and Mathis Rocton. IPEC 2023.
- A note on locating sets in twin-free graphs, with Nicolas Bousquet, Quentin Chuet, Victor Falgas-Ravry, and Amaury Jacques. Discrete Mathematics 2025.
- On the parameterized complexity of computing good edge-labelings, with Davi de Andrade, Júlio Araújo, Ignasi Sau, and Ana Silva. Submitted to a journal.
- **Vertex identification to a forest**, with Ignasi Sau and Dimitrios M. Thilikos. Discrete Mathematics 2026.
- **Graph modification of bounded size to minor-closed classes as fast as vertex deletion**, with Ignasi Sau and Dimitrios M. Thilikos. ESA 2025.
- **Excluding Pinched Spheres**, with Evangelos Protopapas, Dimitrios M. Thilikos, and Sebastian Wiederrecht. Submitted to a journal.
- When does FTP become FPT?, with Matthias Bentert, Fedor V. Fomin, and Petr A. Golovach. WG 2025.
- Fault-Tolerant Matroid Bases, with Matthias Bentert, Fedor V. Fomin, and Petr A. Golovach. ESA 2025.
- **H-Planarity and Parametric Extensions: when Modulators Act Globally**, with Fedor V. Fomin, Petr A. Golovach, and Dimitrios M. Thilikos. Submitted to a conference.
- Faster Algorithms for the Pre-Assignment Problem for Unique Minimum Vertex Cover, with Marthe Bonamy, Timothé Picavet, and Alexander Scott. Submitted to a conference.